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Graph theory and their application

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Abstract

A framework is offered here after reviewing many works based on graph theory. Part one of this chapter surveys the relevant literature on the subject of communication networks and graph theory, while part two offers' instances to demonstrate the connection between the two. By using graph theory to the investigation of electrical networks and deducing their behaviour, this study contributes to the current corpus of knowledge.

Keywords: Networks, graph theory, demonstrate, framework, electrical networks

Introduction

An important first step is to understand that graphs are mathematical structures that provide a visual representation of commonplace problems. Furthermore, network theory permits both the simple representation of a single kind of interaction and the multiple representation of a more complex set of relationships. Notable people like Mark Zuckenberg, Facebook's creator, have used the term "social graphs" to describe the web of ties and connections formed by the network's users. Computer science and other disciplines can benefit from graph theory, a branch of mathematics. It relies on analytical and practical mathematics. By doing so, it is able to incorporate multiple ideas.

Graph theory is a vital resource for contemporary mathematicians. Attempts to comprehend games of chance inspired much of the early work in probability theory, which is quite comparable to the history of graph theory. The study of games and recreational mathematics has inspired a large chunk of graph theory. Leonard Euler published a paper in 1736 that is considered the foundation of graph theory. Famously, he found a solution to the seven bridges problem at Königsberg. This laid the groundwork for the idea of an Eulerian graph in subsequent years. Nothing further was accomplished in this domain for the subsequent hundred years.

For use in electrical networks, Gustuv R. Kirchhoff refined the tree theory in 1847. Arthur Cayley, one of the pioneers of graph theory, was captivated by it for the purpose of counting trees. In 1857, he tried to keep track of structural isomers of saturated hydrocarbons CkH2k+2. To illustrate the CkH2k+2 molecules, he made use of a connected graph. The degree of the vertex representing a carbon atom was four, whereas the degree of the vertex representing a hydrogen atom was one, in accordance with their chemical valences. The sum of all the nodes in this type of network is Here is the formula for n: 3k + 2

Materilas and Methods

Dhananjay Kumar *et al.* (2023) [1] Academic genealogy graphs illustrate the origins and development of fields by recording details about the family trees of scholars and the information transfer from mentors to mentees. This research delves into the Shodhganga academic genealogy graph/network (AGN), a repository for Indian ETDs. We built the Shodhganga-AGN and used topological metrics from the literature on general graphs and genealogical networks to analyse it after we removed potential confusion between the researchers' names in Shodhganga. These metrics have allowed us to identify the schools and professors in India that have had a significant impact on the development of the country's contemporary system of higher learning. The Shodhganga-AGN component is the most densely linked one, with 1,356 researchers and 1,437 advisor-advisee links. Scientists and academics from three main universities make up the bulk of the component's members. When looking for supervision patterns in the genealogy network, we also looked at subgraphs and discovered that majority of them linked scholars from the same institution or field. Our research thus represents an in-depth examination of the academic pedigree of academics listed in Shodhganga and a snapshot of India's research environment spanning decades, as shown through the official advisor-advisee links in Indian universities.

B. Abhishek *et al*. (2022) [2] Using the social network to analyse public data from person-to-person communication destinations could yield interesting findings and information for the overall evaluation of almost any service, product, or behaviour. Since many customers appear to express their opinions on social networks, information mining from these platforms is one of the greatest and most accurate ways to get public conception cues. Technological advancements on the Internet have found out ways to increase participation in activities like blogging, labelling, posting, and informal online discussion. Mining these massive data sources to assess the opinions is, hence, becoming more popular. Social network analysis (SNAI) is a way of studying social designs that uses graph hypotheses and networks. It takes into account various methods for studying the structure of informal organisations and theories that try to explain the framework's components and patterns. It is fundamentally an interdisciplinary field because it has its roots in sociopsychology, statistics, and graph theory. In this chapter, we will go over the SNA hypothesis, as well as the graph hypothesis and data spread, which are introduced briefly. Following that, we'll go over Python's function in SNA, and then we'll construct and recommend informal communities using real Pandas and text-based datasets.

Jesús Rogel-Salazar (2020) [3] The study of relationships between entities that are interdependent is what network analysis is all about. It relies on graphs to depict those things and the relationships between them. Network analysis is useful in many fields, including communications, economics, biology, and statistical physics. The original network used to assign a value and a colour to each node based on how many scenes the character spoke in. Being able to recognise different kinds of patterns and structures is fundamental to our humanity. That is also undeniably accurate when considering the interpersonal dynamics between nations, groups, and organisations. The goal of several approaches is to find community structures in networks. These include generative model techniques, optimisation, hierarchical clustering, and graph partitioning. Several methods exist for finding communities within a network.

Stefan M Kostić *et al*. (2020) [4] The telecommunications industry is tremendously competitive, thus telco operators must constantly be on the lookout for new information on their customers' habits and preferences. That is where graph theory and social network analytics come in handy. Using measures like in/out degree, authority, eigenvector, impact (first and second order), and hub values, this research analyses a social network represented by a large telecom network graph and groups its nodes into clusters. We demonstrate in this research that it is possible to identify and use key nodes in our social network (graph) for churn prediction. We demonstrate that when a monitored telco operator loses this type of node, customers who regularly engage with it are also more likely to leave the operator's network. Therefore, we can anticipate which customers are likely to churn by looking at past call patterns and current churn rates. Top decile lift indicators are used to quantify the results of the churn forecast. The suggested approach is broadly applicable, making it easy to implement in any domain where the presence of homophilic or friendship-based ties is suspected to play a role in churn.

Wojciech Golędzinowski *et al.* (2023) [5] Social network analysis (SNA) is an effective interdisciplinary field that studies the patterns and dynamics of relationships between individuals, groups, and even societies. This article gives a general introduction to support vector analysis (SNA), discussing its origins in graph theory and demonstrating its many uses in domains like sociology, computer science, business, and epidemiology. By examining the theoretical foundations of SNA and its practical implementations, this article aims to demonstrate the importance of SNA in understanding social structures, information diffusion, impact dynamics, and collective behavior. In addition, the article discusses the methodologies and tools used in SNA research, including data collection, network visualization and network metrics. Through a comprehensive analysis of SNA techniques and their applications, this article contributes to the growing knowledge in the analysis of social networks and encourages further exploration of this rich field.

Applications of graph theory

Graphs in Markovian process

One way to study how a variable is currently changing is with Markov analysis. Markov analysis has been utilised as a management tool in recent years, primarily as a marketing aid for studying and forecasting customer behaviour in terms of brand loyalty and switching patterns.

You can think of the transition probability pij as the chance that the system will go from state Ei to state Ej at some point in the future. Whenever a new result or outcome occurs, the process is said to have advanced by one step. Every step signifies a specific time period or condition that leads to a different state.

A rectangular array that summarises the transition probabilities for a certain Markov process is called a state-transition matrix. In this matrix, each row represents the system's current state, and each column represents a potential transition between those states.

Consider a stochastic process with a state Ei and a transition probability pij from Ei to Ej, where Ej is the next state in the process. Following that, we have the one-stage state-transition matrix P.

Table 1: Transition matrix

$$P = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{bmatrix} 0 & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ p_{31} & 0 & p_{33} \end{bmatrix} \end{array}$$

As an example, let's say that Ei is the ith state of a stochastic process and pij is the probability of transitioning from Ei to Ej in one step. Following that, we have the one-stage state-transition matrix P.

The probability distribution for a single step transition to another state is represented by each row in the transition matrix.

$$p_{i1} + p_{i2} + \text{------} + p_{im} = 1, \forall i \quad \text{and } 0 \leq p_{ij} \leq 1$$

A diagram of the state transition matrix is called transition diagraph or weighted diagraph or transition diagram.

A transition diagram illustrates the probabilities of transitioning between different states in any given scenario.

The arrow in the diagram shows the potential states that a process can transition to from the current state. Such a diagraph is given in Fig. 1. and corresponds to the transition matrix in Table 1.

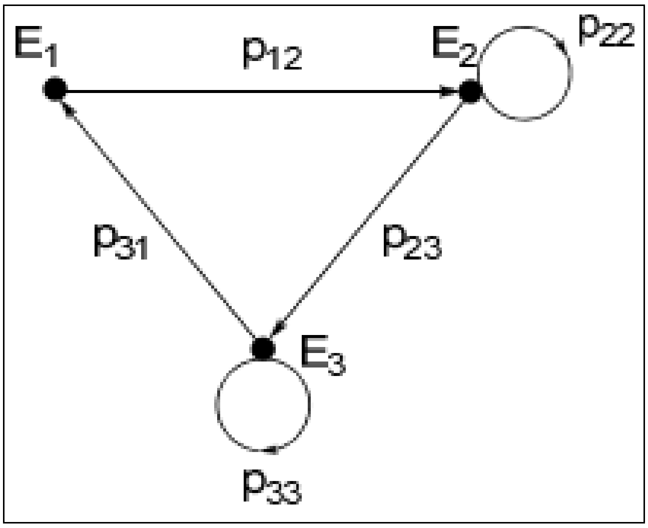

Fig 1: Transition diagraph

Graphs in Network Analysis

Construction, maintenance, fabrication, buying, computer system installation, and R&D designs are just a few examples of the many project kinds that might benefit from the use of Network Scheduling. In order to minimise problems like bottlenecks, delays, and interruptions, this method entails finding and fixing critical components.

A network is a graphical depiction of the processes and activities that make up a project. An arrow with events at its ends represents activity.

Graphs in Biological Science

A recent application of graphs is in biological science. Here diagraphs, cyclic graph and hypergraphs are used to represent biological phenomena. The nodes of the digraph represent entities, while the edges symbolise biological interactions between the nodes, such as transformation, catalysis, complex function, or any other biological process.

Graphs in Groups

A strongly connected digraph with n vertices can be used to represent a group of order n. The diagraph is structured such that each vertex represents an element of the group and each edge is labelled with a generator of the group. Every edge in the directed circuit of an n-vertex graph of an n-order cyclic group has the same label. A group's diagraph, which shows the relationship between element products and directed edge sequences, is an essential tool for group definition. This diagram, called the Cayley diagram, is quite handy for visualising and studying abstract groups. Graph theory is crucial in coding theory due to its use of groups.

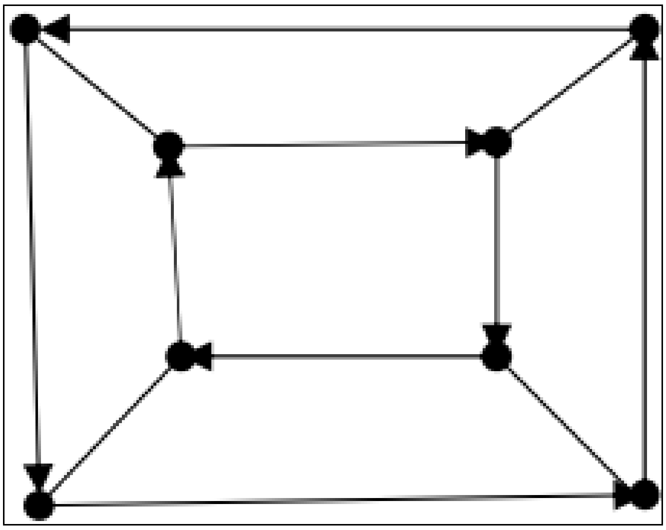

Fig 2: Cayley diagram

Applications of graph theory in electrical networks

A personal computer is required to see the resulting graphs. Electronic parts like capacitors, inductors, resistors, and the like form a circuit when they are connected to one another. The study of network topology is known as electric network evaluation and synthesis. A diagrammatic representation of an electric network problem is possible. A current can be sent from the negative end of the power source to the load when the switch is turned on, completing the electrical circuit. The circuit consists of a source, wires, and a switch. At this point, the electrical network is transformed into a directed graph G with two variables, voltage and current, assigned to each node ek. This chapter offers a few practical ways that graph theory might be used to electrical work analysis and design.

Draw graph from electrical circuit

The graph was created using the circuit diagram. Take a look at Figure 3 for the circuit schematic.

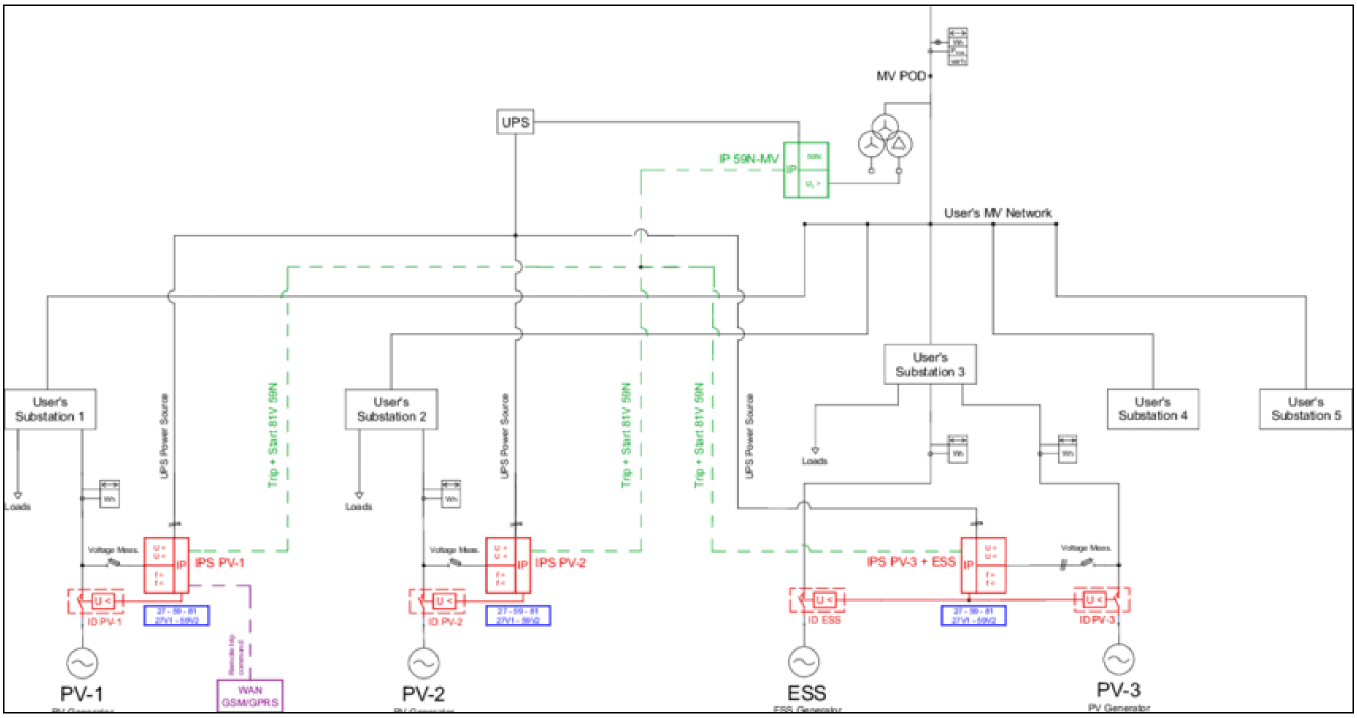

Fig 3: Circuit diagram of electrical network

This circuit has number of branches and that having the load in the form of resistors, capacitors, inductor and a source. All the nodes with corresponding load are enough to draw the graph. The nodes are labelled A, B, C, D, and E in the diagram above. You may see that in figure 4 down below.

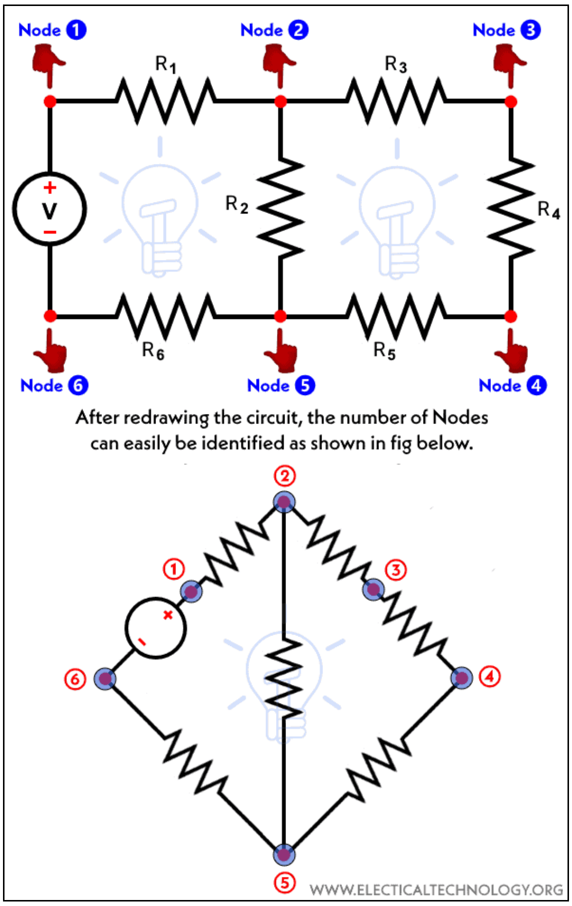

Fig 4: Circuit diagram of electrical network specifying nodes

It is sufficient to create the network using all the nodes with the relevant load.

The schematic served as the basis for the graph. The circuit schematic may be seen in Figure 5.

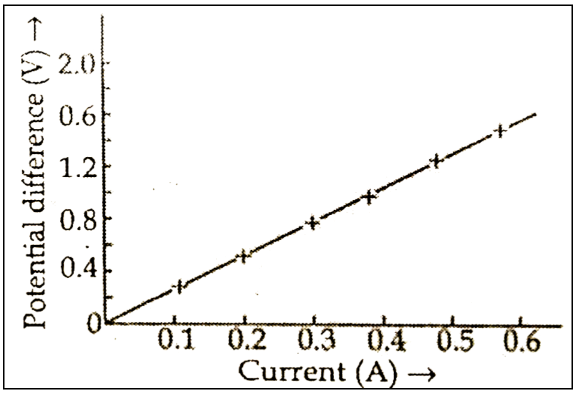

Fig 5: The graph of the circuit diagram

Number of Possible Trees in an Electrical Network

The maximum number of possible trees is determined by the circuit structure. Figure 6 shows the circuit design for finding the possible trees.

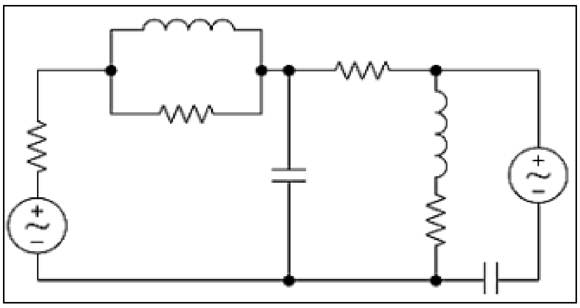

Fig 6: Circuit diagrams for finding the trees of an electrical network

To locate the nodes in an electrical network, one can refer to the circuit diagrams presented in Figure 6. The graph of the above electrical circuit shows in figure 7,

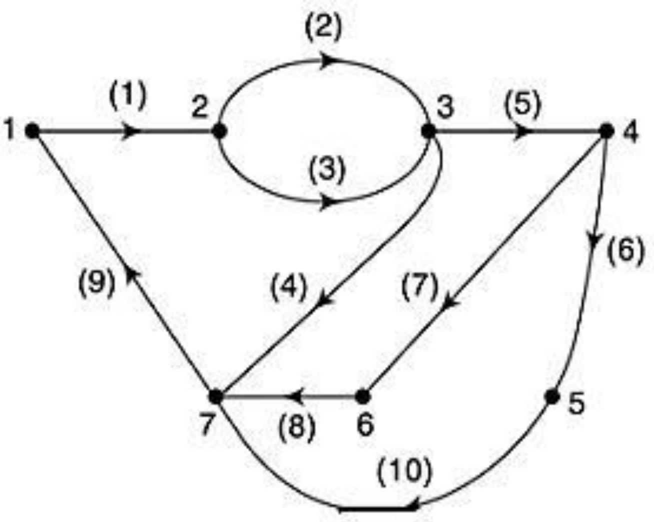

Fig 7: Graph of the above electrical circuit

Figure 8 displays the graph's tree structure.

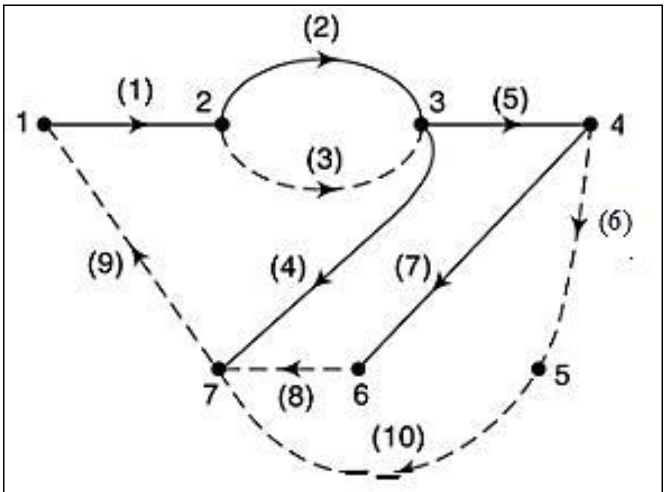

Fig 8: The tree structure of the graph

NODES		BRANCHES									
		1	2	3	4	5	6	7	8	9	10
	1	1	0	0	0	0	0	0	0	-1	0
	2	-1	1	1	0	0	0	0	0	0	0
A =	3	0	-1	-1	1	1	0	0	0	0	0
	4	0	0	0	0	-1	1	1	0	0	0
	5	0	0	0	0	0	-1	0	0	0	1
	6	0	0	0	0	0	0	-1	1	0	0
	7	0	0	0	-1	0	0	0	-1	1	-1

$$\text{The incidence matrix } A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}$$

The reduced incidence matrix was generated by removing the seventh node from the network. When the incidence drops, the matrix becomes into.

$$A_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

Afterwards, the total number of potential graph trees $t = det\,[A_C\,A_C^T]$

$$t = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \right\}$$

$$= \det \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 & 0 \\ 0 & -2 & 4 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$

t = 12

As a result, twelve trees can be considered for the given graph.

The reduced incidence matrix was generated by removing the seventh node from the network.

Graph theory applications

A graph was used to transform any electrical network circuit design according to certain parameters. The following are listed,

1. Remember to leave out the passive element when drawing a graph that has a voltage source in parallel with it.
2. Since the active component and the passive one are shown as being connected in parallel on the graph, this arrangement is technically an open circuit.

Theorem 1: Assuming G is linked, the circuit matrix $B = B_2 B_f$. where matrix B_2 comprises both the cut matrix and 0s and ±1s $Q = Q_1 Q_f$. in which a matrix Q_1 comprises 0s and ±1s.

Proof. First, think about how G is linked. Select a G spanning tree T and reorganise G's edges m. Here, T-branches appear first, while T-links appear last.

Then

$$Q_f = (I_{n-1} \mid Q_{fc}) \text{ and } B_f = (B_{ft} \mid I_{m-n+1})$$

Similarly, The blocks of B can be expressed as,

$$B = (B_1 \mid B_2)$$

Where, Q_{fc} is the submatrix of Q_f.

B_{ft} is the submatrix of B_f.

$$O = B_f Q_f^{\ T} \qquad \text{(Since } BQ^T = 0\text{)}$$
$$= \left((B_{ft} \mid I_{m-n+1})(I_{n-1} \mid Q_{fc})\right)^T$$
$$= \left((B_{ft} \mid I_{m-n+1})(I_{n-1} \mid Q_{fc}^{\ T})\right)$$
$$= B_{ft} I_{n-1} + I_{m-n+1} Q_{fc}^{\ T}$$
$$= B_{ft} + Q_{fc}^{\ T}$$

Similarlly, Q_f is the submatrix of Q, use the same theorem

$$O = BQ_f^{\ T}$$
$$= \left((B_1 \mid B_2)(I_{n-1} \mid Q_{fc})\right)^T$$
$$= \left((B_1 \mid B_2)(I_{n-1} \mid Q_{fc}^{\ T})\right)$$
$$= B_1 I_{n-1} + B_2 Q_{fc}^{\ T} = B_1 - B_2 B_{ft}$$
$$\therefore B_1 = B_2 B_{ft}$$

Hence, $B = (B_2 B_{ft} \mid B_2)$

$$= B_2(B_{ft} \mid I_{m-n+1})$$
$$= B_2 B_f$$

Q may be stated using the aforementioned manner as $Q = Q_1 Q_f$.

Conclusion

The rapidly expanding field of graph theory in mathematics may hold the key to solving a great deal of practical issues. Graph theory and its applications are introduced in a general way in the first chapter of the thesis. The key points of discussion revolve around the graph's practical uses and how it is used in research-oriented difficulties.

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