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Analysis of Fingering Phenomena in Fractured Porous Media Considering Interface Saturation Variation and Capillary Pressure Effects

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Abstract

This paper studies fingering phenomena in fractured porous media. A non-linear equation for water saturation is derived and simplified using similarity transformation. The solution is obtained using a power series method. Convergence is discussed using boundedness assumptions, and an approximate solution is obtained by truncating the series. Results show that saturation increases with time and distance, which agrees with physical behavior with the effect for some interesting choices of parametric data value and appropriate boundary condition and graphical and numerical presentation.

Keywords: Fingering Phenomenon, Capillary pressure, Similarity Transformation, Fractured Porous Media, Power Series Method

Introduction

An oil reservoir consists of a porous material, with its pores filled with various hydrocarbon substances, commonly referred to simply as "oil. " The heterogeneous nature of the porous medium indicates that properties of the rock, such as porosity and permeability, can differ significantly across various locations. The oil fields known as "fractured oil fields" are the most diverse, comprising clusters of porous medium blocks that are divided by a network of fractures. In the initial recovery stage, oil is drawn from the reservoir without any outside help. The subsequent recovery stage involves the retrieval of leftover oil that is held in the porous structure by introducing different fluids like water, gas, steam, or polymers. Issues, such as fingering, occur because of the pressure from the injected fluids, while differences in viscosity and wetting properties between oil and water lead to the development of these issues at their boundary. The injected water moves swiftly through the porous material along networked capillaries that resemble fingers; thus, this occurrence is known as the fingering phenomenon. Many researchers have discussed this phenomenon in homogeneous and heterogeneous porous medium from various point of view. Vyas N. B *et al.* [14] have discussed the magnetic field effect on saturation of injected water with or without magnetic field effect by applied the power series method. Mistry P.R *et al.* [8] have presented mathematical model by use of mass flow rates of oil water in equation of continuity and no linear differential equation for obtained saturation of injected water. Patel S.S [17, 18] Studied in viscous fingering with perturbation Technique. Borana *et al.* [4] applied the Crank-Nicolson finite difference scheme for solution of saturation of water in homogeneous porous medium. Scheidegger [19] has provided an overview of the instabilities associated with displacement fronts in porous materials and highlighted the most relevant technique for oil extraction in the field of reservoir engineering, it is important to keep the fingers steady. Verma [2] examined the statistical characteristics of fingering that happen during a displacement process in a heterogeneous porous medium affected by capillary pressure. Patel Kajal *et al.* [10] studied Homotopy analysis method for saturation of injected water in inclination or without inclination during instability phenomena occurred in secondary oil recovery process in inclined porous media. Patel Kaushal *et al.* [11] demonstrate the analytical method to analysis saturation of water decrease with an increase in injected fluid and time. Patel *et al.* [13, 15] An approximate solution of fingering phenomenon in heterogeneous porous medium with mean pressure explained and fingering phenomena in fluid flow through fracture porous media with inclination and gravitational effect and investigate the applicability of Adomian decomposition method. More Pratiksha *et al.* [9] discussed the phenomena in vertical downward direction in heterogenous porous medium. Meher *et al.* [7] developed the analytical solution of instability phenomenon by Exponential self-similar solution technique with the presence of capillary pressure effect. In previous study, Patel T.R *et al.* [13, 12] have discussed approximate solution water saturation at common interface of instability and fingering phenomena in heterogeneous and homogeneous porous medium with use of power series expansion method. Borana R.N *et al.* [5] also derived solution of instability phenomena in heterogeneous porous medium with help of Crank- Nicolson finite difference method and basic parameters is studied to observe the corresponding effect on saturation of water. Our present effort was focused on use of some different choice of parametric data value and studied the saturation of injected water when water is injected at common interface.

Mathematical Description

Certain assumptions were considered when mathematically formulating this phenomenon. The phenomenon of instability in heterogeneous porous media has been studied based on fundamental assumptions: two immiscible fluids, water and oil, flow incompressible; the porous medium is isotropic, incompressible, and finite, with a horizontal and impermeable base; mass conservation holds, and gravitational forces are ignored. The natural oil reservoir is large enough to evaluate the impact of water injection (water flooding) on the oil extraction process. To investigate the instability phenomenon in heterogeneous porous media, we chose a cylindrical segment of porous matrix with length L (Figure. 1), where three surfaces are impermeable except for one end, where water is introduced. The reliability of the water flood depends on the mobility ratio of oil to water, the variability of the porous medium, fluid separation within the reservoir, and the loss of fluid fronts caused by capillary pressure. These frontal instabilities are usually marked by many penetrating extensions of the displacing fluid. As a result, all oil at the starting boundary x = 0 (with x measured along the direction of movement) is shifted a distance L because of water injection [5].

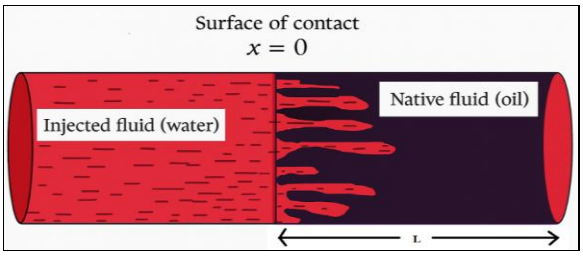

Fig 1: A cylindrical heterogeneous porous matrix is used to illustrate Fingering Phenomenon.

Mathematical Formulation

For the double phase flow system, we may express the seepage velocities of the wetting phase and the non-wetting phase by assuming the validity of Darcy's law [3, 21].

$$V_w = -\frac{k_w}{r_w} K \frac{\partial P_w}{\partial x} \tag{1}$$

$$V_o = -\frac{k_o}{r_o} K \frac{\partial P_o}{\partial x} \tag{2}$$

In this context, *K* represents the variable permeability of the fractured porous medium, while k_w and k_o denote the relative permeability of the displacing fluid and the native fluid, respectively. P_w and P_o represent pressures; r_w and r_o denote the constant kinematic viscosities of the displacing and native fluids respectively, the equations of continuity for these two fluids are expressed as,

$$\emptyset \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \tag{3}$$

$$\emptyset \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \tag{4}$$

Where, $\emptyset = \emptyset(x)$ is the porosity of the fractured porous media. The logical direct relationship between capillary pressure and phase saturation can be written as, [19, 20].

$$S_w + S_o = 1 \tag{5}$$

$$P_c = P_o - P_w \tag{6}$$

We consider the capillary pressure and relative permeability function for water as [6],

$$P_c = \beta (S_w^{-\frac{1}{2}} - C_0) \tag{7}$$

Where, β and C_0 are constant of proportionality. Corey's Model, utilized in this paper, provided the standard correlation among relative permeability and phase saturation [6].

$$k_w = S_w^3$$
$$k_o = 1 - \alpha S_w \tag{8}$$

Where, a is constant and its value is α=1.11 Since, The porosity and permeability of fractured porous media [1],

$$\emptyset(x) = \frac{1}{b_1 - b_2 x} \tag{9}$$

$$K(x) = K_c \emptyset(x) \tag{10}$$

Where, $b_1 - b_2 x \geq 1$; where, a and b are constants, for saturation, Substituting is the key to obtaining the Eq. (1) and Eq. (2) in continuity of the equations respectively, we get,

$$\emptyset \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k_w}{r_w} K \frac{\partial P_w}{\partial x}\right) \tag{11}$$

$$\emptyset \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x}\left(\frac{k_o}{r_o} K \frac{\partial P_o}{\partial x}\right) \tag{12}$$

Eliminating $\frac{\partial P_w}{\partial x}$ from Equation (11) and Equation (6), we have

$$\emptyset \left(\frac{\partial S_w}{\partial t}\right) = \frac{\partial}{\partial x}\left[\frac{k_w}{r_w} K \left(\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x}\right)\right] \tag{13}$$

Now From, Eq. (12) and using Eq. (13) we derive,

$$\frac{\partial}{\partial x}\left[\left(\left[\frac{k_w}{r_w} + \frac{k_o}{r_o}\right] K\right) \frac{\partial P_o}{\partial x} - \frac{k_w}{r_w} K \frac{\partial P_c}{\partial x}\right] = 0 \tag{14}$$

Integrating Eq. (14) we get,

$$\left[\frac{k_w}{r_w} K \frac{\partial P_c}{\partial x} - \left(\left[\frac{k_w}{r_w} + \frac{k_o}{r_o}\right] K\right) \frac{\partial P_o}{\partial x}\right] = V \tag{15}$$

Where, V is the constant of Integration. From Eq. (15)

$$\frac{\partial P_o}{\partial x} = \frac{V}{\left[\frac{k_w}{r_w} + \frac{k_o}{r_o}\right] K} + \frac{\frac{k_w}{r_w} K}{\left[\frac{k_w}{r_w} + \frac{k_o}{r_o}\right] K} \frac{\partial P_c}{\partial x} \tag{16}$$

By putting the values of $\frac{\partial P_o}{\partial x}$ in Eq. (13) it gives,

$$\emptyset\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial x}\left[\frac{\frac{k_o}{r_o}}{\left(1+\frac{k_o r_w}{r_o k_w}\right)}K\frac{\partial P_c}{\partial x}-\frac{V}{\left(1+\frac{k_o r_w}{r_o k_w}\right)}\right]=0 \tag{17}$$

The value of the pressure of oil (P_o) can be written as,

$$P_o=\bar{P}+\frac{1}{2}P_c \tag{18}$$

Where, $\bar{P}$ is a constant that signifies the average pressure. Now

$$\left(\frac{\partial P_o}{\partial x}\right)=\frac{1}{2}\left(\frac{\partial P_c}{\partial x}\right) \tag{19}$$

Substituting the value of $\frac{\partial P_o}{\partial x}$ in Eq. (15) it gives,

$$V=\frac{1}{2}K\left[\frac{k_w}{r_w}-\frac{k_o}{r_o}\right]\left(\frac{\partial P_c}{\partial x}\right) \tag{20}$$

Substituting the Eq. (19) into Eq. (17), We obtain,

$$\emptyset\left(\frac{\partial S_w}{\partial t}\right)+\frac{1}{2}\frac{\partial}{\partial x}\left[\frac{k_w}{r_w}K\frac{dP_c}{dx}\frac{\partial S_w}{\partial x}\right]=0 \tag{21}$$

Replacing this Value Eq. (7), (8) and Eq. (10) into Eq. (20), We get,

$$\emptyset\left(\frac{\partial S_w}{\partial t}\right)-\frac{1}{4}\frac{\beta K_c}{r_w}\frac{\partial}{\partial x}\left[\emptyset S_w^{\frac{3}{2}}\frac{\partial S_w}{\partial x}\right]=0 \tag{22}$$

This partial differentiation-based nonlinear equation represents the absorption process of Instability phenomena and demonstrates the equation of motion that saturates the liquid injected into a fractured porous medium.

Solution of the Problem

Dimensionless Variable is chosen as follow,

$$X=\frac{x}{L} \quad T=\frac{1}{4}\frac{\beta K_c}{L^2 r_w}t \tag{23}$$

Replacing this Value in the Eq. (21), we have,

$$\frac{\partial S_w}{\partial T}-\left(\frac{\partial S_w^{\frac{3}{2}}}{\partial X}\right)\frac{\partial S_w}{\partial X}-\frac{1}{\emptyset}\frac{\partial\emptyset}{\partial X}S_w^{\frac{3}{2}}\left(\frac{\partial S_w}{\partial X}\right)-S_w^{\frac{3}{2}}\left(\frac{\partial^2 S_w}{\partial X^2}\right)=0 \tag{24}$$

Eq. (23) This is a non-linear partial differential equation. For the purpose of simplifying the problem, we will focus on the term $\frac{1}{\emptyset}\frac{\partial\emptyset}{\partial X}$ has been simplified as,

$$\frac{1}{\emptyset}\frac{\partial\emptyset}{\partial X}=\frac{\partial}{\partial X}(\log\emptyset)=L\left(\frac{b_2}{b_1}\right)=\frac{L}{2\sqrt{T}}\,(neglacting\ the\ higher\ term\ of\ X) \tag{25}$$

Where, $\frac{b_2}{b_1}=\frac{1}{2\sqrt{T}}$ for any $0<T\leq 1$ Hence, Eq. (23) will be,

$$\frac{\partial S_w}{\partial T} - \left(\frac{\partial S_w^{\frac{3}{2}}}{\partial X}\right)\frac{\partial S_w}{\partial X} - \left(\frac{L}{2\sqrt{T}}\right)S_w^{\frac{3}{2}}\left(\frac{\partial S_w}{\partial X}\right) - S_w^{\frac{3}{2}}\left(\frac{\partial^2 S_w}{\partial X^2}\right) = 0 \tag{26}$$

The Appropriate Boundary Condition to Solve Eq. (25) as,

$$S_{wo}(X,0) = 0 = S_{wc}, \quad for \ X > 0 \tag{27}$$

$$S_{wo}(0,T) = 1 = S_{wo}, \quad for \ T > 0 \tag{28}$$

$$S_{wo}(1,T) = 0 = S_{wi}, \quad for \ T > 0 \tag{29}$$

Choose the Similarity Transformation [16],

$$S_{wo}(X,T) = f(\zeta) \qquad where, \zeta = \frac{X}{2\sqrt{T}} \tag{30}$$

The governing equation (25) simplifies to a standard ordinary discrimination equation, accompanied by the boundary conditions (26) and (28) that we retain,

$$f(\zeta)^{\frac{3}{2}} f''(\zeta) + \frac{3}{2} f(\zeta)^{\frac{1}{2}} f'^2(\zeta) + L f(\zeta)^{\frac{3}{2}} f'(\zeta) - \zeta f'(\zeta) = 0 \tag{31}$$

With the Boundary Conditions Eq. (26) and Eq. (27),

$$f(0) = S_{wo}, \quad X = 0, T > 0 \tag{32}$$

$$f(\infty) = S_{wc}, \quad T = 0, X > 0 \tag{33}$$

$$f'^{(0)} = \vartheta, \quad for \ any \ T > 0 \tag{34}$$

To determine the successive coefficients of the Maclaurin series at $\zeta=0$. By computing the nth derivative of Equation (30), and then solving for $f^{n+2}(\zeta)$ and evaluating at $\zeta=0$, we have,

$$\sum_{m=0}^{n} \binom{n}{m} \left(\frac{d^m}{d\zeta^m} f(\zeta)^{\frac{3}{2}}\right) f^{(n-m+2)}(\zeta) =$$

$$-\frac{3}{2}\sum_{m=0}^{n} \binom{n}{m} \left(\frac{d^m}{d\zeta^m} f(\zeta)^{\frac{1}{2}}\right)\left(\frac{d^{n-m}}{d\zeta^{n-m}} (f'(\zeta))^2\right) \tag{35}$$

$$-L\sum_{m=0}^{n} \binom{n}{m} \left(\frac{d^m}{d\zeta^m} f(\zeta)^{\frac{3}{2}}\right) f^{(n-m+1)}(\zeta)$$

$$-nf^{(n)}(\zeta) - \zeta f^{(n+1)}(\zeta)$$

To solve this, we must get the derivatives of $f^n(0)$ for all $n = 1,2,3,\dots$, $f'(0)$can be obtained from Eq. (32) and formula Eq. (30) can be used to find the derivative $f''(0)$. By entering $n = 1,2,3,\dots$, all additional higher derivatives can also be found using formula (34). Consequently, the intended worth of $f(\zeta)$ can be calculated using Maclaurin's series,

$$f(\zeta) = \sum_{j=0}^{\infty} f^j(0)\frac{\zeta^j}{j!} \tag{36}$$

Therefore, we have,

$$f(\zeta) = f(0) + \zeta f'(0) + \frac{\zeta^2}{2!} f''(0) + \frac{\zeta^3}{3!} f'''(0) + \frac{\zeta^4}{4!} f^{iv}(0) + \cdots \tag{37}$$

$$S_w(X,T) = f(0) + \frac{X}{2\sqrt{T}} f'(0) + \frac{X^2}{8(\sqrt{T})^2} f''(0) + \frac{X^3}{48(\sqrt{T})^3} f'''(0) + \frac{X^4}{384(\sqrt{T})^4} f^{iv}(0) + \frac{X^5}{3840(\sqrt{T})^5} f^{v}(0) + \cdots \tag{38}$$

Now From Eq. (28),

Where,

$$f(0) = S_{wo}$$

$$f'(0) = \vartheta$$

$$f''(0) = -3\left[\frac{\vartheta^2}{S_{wo}} + L\vartheta\right]$$

$$f'''(0) = -\left[\frac{3}{4}\frac{\vartheta^3}{S_{wo}^2} + \frac{9}{2}\frac{\vartheta f''}{S_{wo}} + \frac{3L}{2}\frac{\vartheta^2}{S_{wo}} + \frac{\vartheta}{S_{wo}^{\frac{3}{2}}} + Lf''\right]$$

$$f^{iv}(0) = -\left[6\frac{\vartheta f^{iv}}{S_{wo}} - \frac{3}{8}\frac{\vartheta^4}{S_{wo}^3} + \frac{9}{2}\frac{f''^{\vartheta^2}}{S_{wo}^2} + \frac{9}{2}\frac{f''^2}{S_{wo}} + \frac{9}{2}\frac{L\vartheta f''}{S_{wo}} + \frac{3L}{4}\frac{\vartheta^3}{S_{wo}^2} + \frac{2f''}{S_{wo}^{\frac{3}{2}}} + Lf'''\right]$$

$$f^{v}(0) = -\left[\frac{15}{2}\frac{\vartheta f^{iv}}{S_{wo}} + \frac{6L\vartheta f'''}{S_{wo}} + \frac{15f''f'''}{S_{wo}} + \frac{9L}{2}\frac{f''^2}{S_{wo}} + \frac{15}{2}\frac{\vartheta^2 f'''}{S_{wo}^2} + \left(\frac{9L}{2} + 9\right)\frac{\vartheta^2 f''}{S_{wo}^2} + \frac{9}{4}\frac{\vartheta f''^2}{S_{wo}^2} - \frac{15}{4}\frac{\vartheta^3 f''}{S_{wo}^3} - \frac{3L}{8}\frac{\vartheta^4}{S_{wo}^3} + \frac{9}{16}\frac{\vartheta^5}{S_{wo}^4} + \frac{3f'''}{f^{\frac{3}{2}}}\right] \tag{39}$$

The equation is when water is added to the general interface processes for recovering secondary oil, such as, in fractured porous environments, Eq. (36) demonstrates that liquid injection saturation takes place during the Fingering phenomena.

Analysis of Convergence

The Convergence of the Maclaurin Series Eq. (36) depends on the growth of the co-efficient $f^j(0)$. From the recurrence relation Eq. (34), Each higher derivatives is expressed as a finite combination of lower order derivatives and polynomial non linearities in *f*. Assume there exists Constant C, M > 0 Such that,

$$\left|f^j(0)\right| \le CM^j\, j! \text{ for all } j \ge 0. \tag{40}$$

Then the general term,

$$u_j = \frac{f^j(0)}{j!}\zeta^j \quad satifies \quad |u_j| \le C(M|\zeta|)^j \tag{41}$$

Now, Apply Cauchy Root Test for convergence,

$$\lim_{j\to\infty} sup|u_j|^{\frac{1}{j}} \le M|\zeta| \tag{42}$$

Therefore, the series converges for $|\zeta| < \frac{1}{M}$. Therefore, The Maclaurin series has a finite ROC given by $R = \frac{1}{M}$ series is locally convergent but not globally.

Saturation $S_w(X,T)$ vs. Distance X

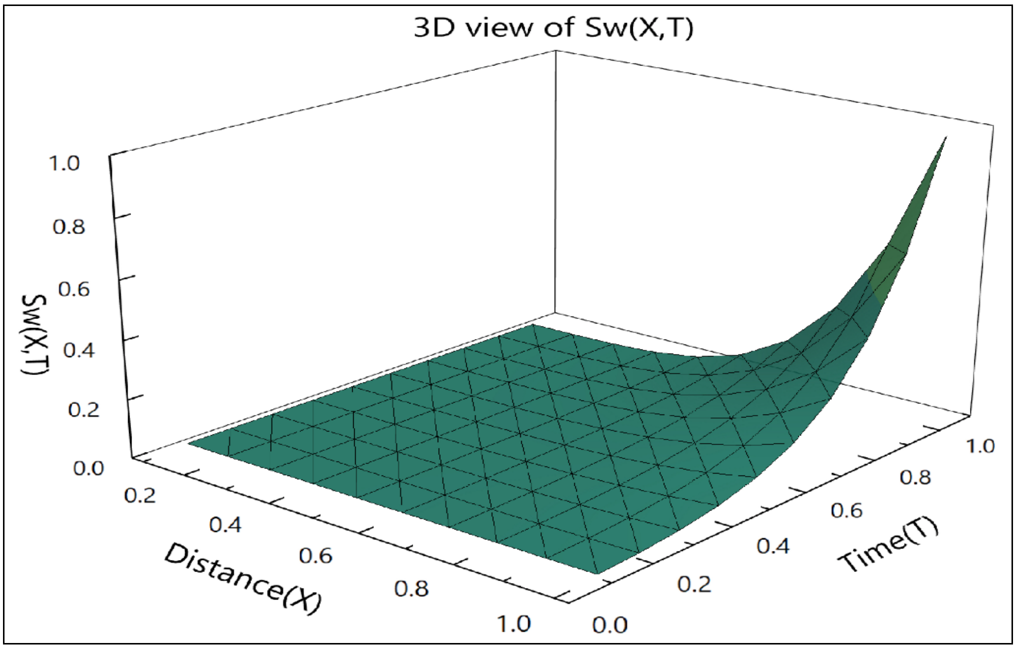

Fig 2: $S_w(X,T)$ when $T=0.1\ to 1$ and $\vartheta=0.01$, $S_{wo}=0.0162$, $L=5$

Table 1: Numerical assessment of the Fingering Phenomenon in Fractured Porous Media.

X	T	T	T	T	T	T	T	T	T	T
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.0162	0.0162	0.0163	0.0163	0.0163	0.0163	0.0163	0.0163	0.0163	0.0164
0.2	0.0162	0.0162	0.0163	0.0163	0.0163	0.0164	0.0164	0.0165	0.0166	0.0167
0.3	0.0162	0.0163	0.0164	0.0165	0.0166	0.0169	0.0172	0.0176	0.0182	0.0190
0.4	0.0162	0.0164	0.0167	0.0171	0.0176	0.0185	0.0197	0.0214	0.0240	0.0276
0.5	0.0163	0.0167	0.0174	0.0185	0.0201	0.0224	0.0260	0.0313	0.0391	0.0504
0.6	0.0164	0.0172	0.0188	0.0212	0.0249	0.0305	0.0390	0.0519	0.0711	0.0991
0.7	0.0166	0.0183	0.0213	0.0261	0.0335	0.0450	0.0628	0.0901	0.1311	0.1913
0.8	0.0170	0.0199	0.0253	0.0339	0.0474	0.0688	0.1025	0.1547	0.2337	0.3501
0.9	0.0175	0.0224	0.0312	0.0456	0.0687	0.1058	0.1650	0.2573	0.3979	0.6059
1	0.0183	0.0259	0.0397	0.0626	0.0998	0.1605	0.2582	0.4121	0.6475	0.9972

Quantitative Results and Analysis

During the secondary oil recovery process, when water is injected, the solution Eq. (37) illustrates the saturation of the injected fluid, which is an ascending power series of X with time T>0. The answer Eq. (36) which is also in terms of power series in ζ, satisfies constraints Eq. (31) and (32) as well. As we have just examined the first five terms in the power series, it provides a rough explanation for the instability phenomenon and also shows that the convergent power series. We saw water saturation from the graph at $X=0$ when the instability phenomena happen at the common interface for a certain value of the parameter. Figure 2 depicts the graph of $S_w(X,T)$ vs distance measured from the common interface in dimensionless variables. At time T=0.1, the saturation graph continuously increases until X=1, where it abruptly increases due to the initial unsaturation of the fractured porous medium. For various values of X, the saturation of water increases consistently while time T grows to T=0.1-1; The Numerical Solution for the governing Eq. (25) representing instability phenomenon in fractured porous media is obtained by using Maclaurin's series given by the Eq. (36) and (37). The term $\frac{L}{2\sqrt{T}}$ that appears in Eq. (25) is referred to as the heterogeneity coefficient for the governing equation, if $\frac{L}{2\sqrt{T}}=0$. It describes the phenomenon of instability in a homogeneous porous medium. The numerical values representing the saturation of water at different time intervals are displayed in Table 1, In conjunction with its graphical depiction from Figure 2, It is evident that the saturation of water rises over time and also increases with distance X. Therefore, the physical reality of the issue is maintained in the scenario of a fractured porous medium, allowing for a significant quantity of oil to be displaced.

Conclusion

The investigation shows water saturation increases uniformly with distance and time, indicating a stable displacement front with negligible viscous fingering. Viscous and capillary forces suppress instabilities, ensuring a smooth saturation profile.

Early-stage, localized fingering in fractures due to permeability and velocity variations diminishes over time without affecting overall stability. Power-series representation confirms gradual saturation rise from $S_{w0}=0.0162$, with minimal interface disturbance. The analytical solution captures initial fingering at X=0, while numerical results show consistent saturation increase with radial distance and time. Permeability, capillary pressure, and injection rate variations in fractured media strongly influence saturation and displacement dynamics. The model improves predictive accuracy, enhances sweep efficiency understanding, and demonstrates stable, uniform flow (Table 1, Figure 2).

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Conflict of Interest
The Authors declared that they have no conflict of interest.

Nomenclature

V_w	Velocity of water in the wetting phase $[m/s]$
V_o	Velocity of the oil in the non-wetting phase $[m/s]$
K	Porous medium's permeability $[m^2]$
k_w	Permeability to the wetting phase $[m^2]$
k_w	Permeability to the non-wetting phase $[m^2]$
P_w	Pressure of the water in the wetting phase $[Pa]$
P_o	Pressure of the oil in the non-wetting phase $[Pa]$
$\emptyset$	The porous medium's porosity (volume of voids/total volume)
S_w	Saturation of the wetting phase (water) (volume of water / volume of voids)
L	characteristic length $[m]$
T	Dimensionless time [Sec]
r_o	oil viscosity
r_w	Water viscosity
P_c	capillary pressure
β	Proportionality Constant

References

1. Shah DA, Parikh AK. Mathematical solution of fingering phenomenon in vertical downward direction through heterogeneous porous medium. Advances in Mathematics: Scientific Journal. 2021;10(1):483–496.
2. Verma AP. Statistical behavior of fingering in a displacement process in heterogeneous porous medium with pressure. Revue Roumaine des Sciences Techniques, Série de Mécanique Appliquée. 1969;14(3).
3. Bear J. On the flow of two immiscible fluids in fractured porous media. In: Developments in Soil Science. Vol. 2. Amsterdam: Elsevier. 1972;2:177–202.
4. Borana *et al*. The solution of instability phenomenon arising in homogeneous porous media by Crank–Nicolson finite difference method. International Journal of Innovative Research in Science, Engineering and Technology. 2014;3(2).
5. Borana *et al*. Instability phenomenon in two-phase heterogeneous porous medium and numerical solution of the governing equation. Journal of Emerging Technologies and Innovative Research. 2019;6(5):198–205.
6. Kesarwani J, *et al*. Mathematical modelling of fingering phenomenon using homotopy analysis method. In: AIP Conference Proceedings. Vol. 2214. Melville (NY): American Institute of Physics; 2020. p. 020029.
7. Meher R, *et al*. Exponential self-similar solutions technique for imbibition phenomenon arising in double-phase flow through porous medium with capillary pressure. International Journal of Applied Mathematics and Mechanics. 2011;7(8):29–40.
8. Mistry P, *et al*. Mathematical model and solution for fingering phenomenon in double-phase flow through homogeneous porous media. The Scientific World Journal. 2013;2013:470174.
9. More P, *et al*. Solution of fingering phenomenon in double-phase flow through heterogeneous porous media for vertically downward direction. International Journal of Mathematics Trends and Technology. 2021;67.
10. Patel KK, *et al*. Application of homotopy analysis method in one-dimensional instability phenomenon arising in inclined porous media. American Journal of Applied Mathematics and Statistics. 2014;2(3):106–114.
11. Patel K, *et al*. Analytical study of instability phenomenon using Laplace transform. Mathematical Modelling of Engineering Problems. 2022;9(3).
12. Patel K, *et al*. Power series solution of fingering phenomenon arising in fluid flow through homogeneous porous media. Applications and Applied Mathematics: An International Journal. 2011;6(2):7.
13. Patel KR, *et al*. An approximate solution of instability phenomenon in heterogeneous porous media with mean pressure.

General Mathematics. 2012;10(2):9–21.
14. Vyas NB, *et al*. Power series solution of instability phenomena in double-phase flow through porous media under magnetic field effect. International Journal of Applied Mathematics and Mechanics. 2011;7(16):1–10.
15. Patel HS, Meher R. Simulation of fingering phenomena in fluid flow through fracture porous media with inclination and gravitational effect. Journal of Applied Fluid Mechanics. 2016;9(6):3135–3145.
16. Mehta MN. Asymptotic expansion of fluid flow through porous media. 1977.
17. Patel SS. A problem arising in viscous fingering phenomenon in two-phase immiscible flow with singular perturbation technique. International Journal for Scientific Research and Development. 2013;1(4):2321–0613.
18. Patel SS. Singular perturbation solution of the problem of viscous fingering phenomenon in multifluid immiscible flow. International Journal of Engineering Technologies and Management Research. 2018;5(4):2454–1907.
19. Scheidegger A. The statistical behavior of instabilities in displacement processes in porous media. Canadian Journal of Physics. 1961;39(2):326–334.
20. Scheidegger AE. The physics of flow through porous media. Toronto: University of Toronto Press; c1957.
21. Verma AP, Mishra. Similarity solution for instabilities in double-phase flow through porous media. Journal of Applied Physics. 1973;44(4):1622–1624.