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Higher-Dimensional Foundations and Cubical Methods in Homotopy Type Theory

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Abstract

Homotopy Type Theory (HoTT) is a powerful theory that brings together computer science, higher dimensional mathematics and topology and logic. The theory is an extension of Martin-Löf Type Theory that includes identity types, higher inductive types and univalence, allowing the mathematical structures to be understood as both homotopical and computational. In this paper, I will explore the shift from the set-theoretic foundation to higher dimensional type-theoretic foundation, focusing on the conceptual importance of the identification types and the univalent foundations. The study also explores homotopy equivalence between intervals, homotopy-equivalent computational interpretations of interval structure and formal proof systems, and cubical type theories. Special emphasis is placed on the links between cubical methods and the formalisation of mathematical reasoning in a theorem prover. The paper also mentions some applications of higher dimensional types in topology, synthetic homotopy theory, patch theories, quotient constructions, and computational mathematics. Homotopy Type Theory combines geometric intuition and logical formalism to give a constructive and machine-verifiable base for modern mathematics. The talk illustrates how cubical approaches are able to address the shortcomings of intensional type theory whilst reinforcing the computational interpretation of equality, equivalence and higher-dimensional structures in current mathematical and computational practice.

Keywords: Homotopy Type Theory, Cubical Type Theory, Univalent Foundations, Higher-Dimensional Types, Identity Types, etc.

Introduction

Homotopy Type Theory (HoTT) has completely changed how we view the connections among logic, topology and computer science. As a new foundation for mathematics, HoTT has added higher-dimensional interpretations to the usual set-theoretic approach to mathematics that is based on collection of elements combined by extensional equality. Set theory has served as the foundation for most of modern mathematics for more than a millennium, but it has significant limitations in its ability to represent higher-dimensional structures, equivalences and constructive computations. HoTT extends Martin-Löf Intensional Type Theory by introducing new types (identity types), priniciples of univalence and patterns for building more complex constructs (higher inductive constructions) out of simpler ones. In HoTT, every mathematical object (i.e. type) is represented with respect to its corresponding proof or construction as so-called computational objects. Rather than relying on classical notions of equality associated with the theory-of-sets, identification between two objects using HoTT is interpreted as a "path/transformation" between those objects. Therefore, we think of a larger hierarchy of identities whereby identities have their own higher-dimensional identities. As such, we now view mathematical structures as, not as independent, separate objects, but mostly as objects that are "connected" by a continuous path of equivalence and homotopy. This conceptualisation creates a clear correspondence between type theory and much of algebraic topology, particularly in terms of the relationship between types and topological spaces.

The development of univalent foundations has solidified the relationship between univalence and computer science. The univalence axiom of Vladimir Voevodsky states that if two mathematical structures are equivalent, they can be treated as the same within formal reasoning. By ensuring that isomorphic structures produce the same outcomes in proofs and computations, the univalence axiom allows for a seamless transition from informal to formal mathematical reasoning. Homotopy type theory creates a setting in which mathematical reasoning can be performed as both constructive and computational, yet still respects the requirements of logic. An important advancement in hoTT is the introduction of cubical type theory that introduces geometric cube-like structures and interval variables to type-theoretically represent paths and homotopies computationally. In cubical type theories, equalities can be represented constructively by geometric transformations, thus resolving the limitations of intensionally typed systems when performing computations. Cubical type theories also allow for effective implementations of the univalence axiom and higher inductive types using proof assistants and theorem provers, thus enhancing the capacity of computers to verify complex mathematical proofs while maintaining computational consistency.

Higher-dimensional type theory uses methods from homotopy theory not just in mathematics but for many other things as well, such as: topology, formal verification, programming language semantics, artificial intelligence (AI), and automated reasoning systems. Using synthetic homotopy theory we can build spaces or topologies in type theories directly without needing to use traditional Euclidean-style coordinate systems or open sets. Higher-dimensional structures are also becoming common in computational representations of quotient spaces, patch theories, and equivalences. This paper will look into the conceptual basis and computational importance of higher-dimensional type theories focusing specifically on cubical homotopy type theory and univalent foundations. The discussion will begin with an explanation of why higher-dimensional structures are replacing set theoretic foundations, moving down the path to cubical modelling, interval reasoning, and (homotopy based) equality. Then it will examine the use of higher-dimensional types in topology, computational mathematics, and mathematical automation. Through this analysis one can see, that homotopy type theory is a new logic system, as well as being a major step towards mathematically, topologically and reasoned by computer science.

Literature Review

Markus Hausmann (2023) [1] This page contains an article that provides a theory for Lie groups based on real proper equivariant stable homotopy. The construction of the object form compact isotropy groups in equivariant cells and the evaluation of equivalences on compact subgroups are what the term "proper" refers to. This method yields equivariant cohomology theories which provide credence to the analogue of a R O (G) R O(G) grade. If there are enough transfers and Wirthmuller isomorphisms to back the theory, we call it "genuine." Assuring that these shown theories of cohomology rely only Their main objective is to focus on the "proper G G -homotopy type," which is determined by fixed points under all compact subgroups. An important physical example of the idea and a potential use of our theory are these infinite discrete groups. For this goal, we link our new genuine equivariant theory to the finiteness properties of geometric group theory. The presence of a finite G G CW-model in the universal space for right G G actions is necessary for our triangulated equivariant homotopy category to include a compact G G sphere spectrum. The spectrum is compact only under certain circumstances.

Ko Honda (2024) [5] In order to prevent a contact manifold from being filled by Liouville and to provide a strong enough condition for the Weinstein conjecture to hold, this research will define the contact class and lay the groundwork for higher-dimensional Heegaard Floer homology. The similarities to the Plamenevskaya invariant of crosswise connections are investigated, as is a near cousin of symplectic Khovanov homology. We can handle several situations with this.

Jonathan Keningson (2024) [7] Getting data with a lot of variables that works well is one of the hardest things for current AI and ML. The huge complexity of the information makes it hard for traditional methods to work in these situations. A lack of visibility into the model's behavior, overfitting, and the curse of dimensionality are among these difficulties. Our method for analyzing high-dimensional data is novel in this work. We will use topological and geometric methods to make the model easier to understand and to get a better sense of its structure. In particular, we will go into Persistent Homology as it pertains to Topological Data Analysis. Using this method, one may learn about the data's global structure by identifying topological features like loops and associated components. To display natural patterns in high-dimensional areas, manifold data structures are fundamental. We demonstrate how several concepts from Riemannian and differential geometry may be used to illuminate this matter. This has been observed by everyone. Modern dimensionality reduction methods like as Principal Component Analysis, t-SNE, and UMAP may be enhanced by using these strong mathematical bases to produce low-dimensional, interpretable representations of high-dimensional data. Model understanding and decision-making may both benefit from these representations. Also included are case examples that demonstrate how these strategies can be used to make complicated models more visible and explain how they operate in practice in AI systems. This is of utmost importance in autonomous systems, healthcare, and financial sectors, among other crucial areas. With this book, you will find case studies.

Mihai Damian (2025) [8] According to the research that I had conducted in the past, I had predicted that the complete space of a fibration over the circle is analogous to each closed orientable monotone Lagrangian submanifold in Cn. The results of my prior studies served as the basis for this as well. As a foundation for my inquiry, I use a topological hypothesis on the universal cover of the Lagrangian to prove that this assertion is valid in dimensions that are more than six. This is the premise for my investigation. It is possible to generalize the reasoning to any symplectic ambient manifold that has a second homology group that vanishes. Something like this is something that is within the range of possibility. This is because the Lagrangian may be displaced by a Hamiltonian isotopy. A further demonstration is provided to demonstrate that a generalization to monotone Lagrangians in CPn is valid.

Murali Krishna Pasupuleti (2024) [10] The book "Infinite Patterns: A Journey into the Heart of Topology" is an intriguing journey into the complicated realm of topology. The book focuses on the use of topology in the understanding of infinite-dimensional spaces and the fascinating patterns that arise within them. The basic ideas of topology are discussed in depth in this chapter. These ideas include separation axioms and the topology of infinite-dimensional manifolds, among other important principles. Additionally, it includes a discussion of the practical consequences of these principles by examining them through the perspective of equivariant topology, generative design in engineering, and the interaction between these concepts and algebraic structures. By combining theoretical study with practical applications, this chapter demonstrates the considerable influence that topological discoveries have had on the development of mathematical sciences and the substantial crossover that these insights have had into the actual process of addressing problems in the real world. Providing a comprehensive overview of the current state and future potential of topological studies, this chapter highlights how topology continues to be an essential component in the larger narrative of scientific advancement. This is accomplished through the detailed analysis of cutting-edge research and foundational theories described in the chapter.

Types Versus Sets: Motivation for Higher-Dimensional Foundations: In his argument, Martin-Lof asserts that ontological type theory encompasses a variety of kinds, one of which is identification types. As a result of the incorporation of identification types into the modification all types of Martin-Lof's 1975 ontological type theory now have its very own iterative identification hierarchy, which provides it with extra dimensions. For the reason that the concept incorporates several forms of identification. On the other hand, further extensions or axioms are necessary in order to guarantee the existence of types that include higher-dimensional structures that are not trivial.

Remark 1.2. We use the word "identification types" to emphasize that they are essentially distinct from equality kinds and to indicate the process of recognizing two separate objects. Other names that are often used are shown in table 1.

Remark 1.3. The misconception that behavioral categories do not possess compelling higher-dimensional structures was not dispelled by iterative identification and was thus unsuccessful. This is due to the fact that behavioral typing almost always validates equality types, the fact that appearances of resemblance between identification types and equality types lead one to assume otherwise, and the fact that equality types are always simple. In spite of this, higher-dimensional structures in behavioral type theories sorts who aren't necessarily equal. The next paragraphs present a few type theories that demonstrate this.

The homotopy type theory is the foundation for this syntax, which incorporates both UniTT+hit is the theory of univalent types and higher inductive types. This idea builds on the premise that came before it, and it incorporates the univalence axiom as well as higher categories of inductive reasoning. For the work that is being done for the thesis, it is employed throughout the process. Univalent type theory is what I'll use instead of homotopy type theory. theory for two significant reasons. The first reason is because I believe it to be more accurate than the alternative explanation that may be given. When it comes to the first place, the term "homotopy type theory" may be understood in a variety of various ways, depending on the person who is speaking and the circumstances that are now taking place. It might be referring to the whole field of study, it could be referring to type theory alone, or it could be referring to type theory with a specific interpretation attached to it; all of these are possible. Furthermore, despite the fact that this interpretation is the most significant component of my thesis, I have seen that type theory has applications in computer science that are not connected to homotopy theory. This is something that I have discovered. It is usual practice to refer to the type theory that is discussed in the appendix of Homotopy Type Theory: Univalent Mathematical Foundations as the HoTT thesis. This is due to the fact that the type theory-related material is located in the appendix. The term HoTT is the standard way to refer to the type theory covered in the appendix of Homotopy Type term: Univalent Foundations for Mathematics. This is due to the fact that the type theory-related material is located in the appendix.

System {=} of homotopy type. The universal theory of univalent types, as UniTT is expanded upon by this type theory that was introduced by Vladimir Voevodsky. It includes an additional layer that incorporates internalized equality. There are two sorts of the structure: fibrant types, which adhere to the principle of homotopy equivalence, and nonfibrant types, which may or may not adhere to there. With the introduction of the new layer of nonfibrant types, it is now feasible to "break" homotopy equivalence and construct, for instance, simplicial types, which would have been physically impossible without this feature. In addition, Daniel R. Grayson has developed a prototype of a proof checker.

System with two levels. A lot of ideas for this type theory came from Vladimir Voevodsky's homotopy type system. Nicolai Kraus, Paolo Capriotti, and Thorsten Altenkirch were the creators. However, the upper (non-fibrant) level of Martin-Lof type theory is modified to an identification type that adheres to the UIP principle. To the best of my knowledge, this type of concept isn't used anyplace, even though I won't be discussing this two-layer structure.

The theory of real-cohesive homotopy type type. The type theory, mainly created by Urs Schreiber and Michael Shulman, uses synthetic topological reasoning to further enhance UniTT. The formal establishment of a new context with crisp variables allowed for the notion of discontinuous mappings to be considered. It is now possible to derive non-homotopical conclusions by using the straightforward reasoning of synthetic homotopy theory. This chance becomes possible in this new setting, thanks to the numerous new modal operators. Unfortunately, no practical application of this idea exists at the present time. Type theory with only two dimensions. The computational aspects of the univalence rules and higher inductive types were left out of this early effort by Robert Harper and Dan Licata. It was their first effort. The type theory was an advance toward ontological cubical type theory, but as of this writing, no implementation of it is known. Nevertheless, how to expand the method to higher dimensions was not apparent back then.

Cubical type theories {□}. In light of recent developments in UTT's cubical set description, many ontological type theories with explicit cubical structures have been proposed. Simon Huber, Cyril Cohen, Andrew M. Pitts, Ian Orton, Anders Mortberg, Valery Isaev, and others are part of this group. Along with them are Thorsten Altenkirch and Ambrus Kaposi. At the assessment level, people may see multi-dimensional patterns. Both a trial application and a formalization are now in development within the NUPRL system. Guarded cubical type theory {⎕,⏲}. The authors Lars Birkedal, Aleš Bizjak, Ranald Clouston, Hans Bugge Grathwohl, Bas Spitters, and Andrea Vezzosi have developed a type theory that combines characteristics of the guarded dependent type theory and the cubical type theories that were previously studied. Among the characteristics that fall within this category are the delays in replacement and subsequent categorizations. In addition to that, a prototype checker is offered.

Cubical Homotopy Type Theory Methods and Univalent Foundations

There will be a summer school in Homotopy Type Theory at Pittsburgh's Carnegie Mellon University. During the course, we talked about cubic type theory and how it works in cubical sets. This part follows that discussion. This is not a study chapter with new results. Instead, it is an introduction to cubical type theory and models for cubical sets. A new set of type theories demonstrates the practical use of Homotopy Type Theory and Univalent Foundations (HoTT/UF). Computers can be used to prove Voevodsky's (2014) univalence premise. Now HoTT/UF has a solid base that can be easily put into use on computers.

The Voevodsky (2014) type theoretic univalence principle says that all forms should be invariant up to equivalence. This is what HoTT/UF is based on. Mathematicians sometimes mix up R and notwithstanding these differences, they are both included in the quotient ring R/ (0). In the minds of computer scientists, two approaches of realising an abstract interface are essentially equivalent. It is official that this method works because it defines these things in terms of type theory. This makes the gap smaller between casual and formal math.

The univalence axiom is important because it solves a number of problems caused by the fact that intensional type theory doesn't have any extensionality rules. Because of univalence, there are things like propositional extensionality (two propositions that are semantically similar) and function extensionality (two functions that are pointwise equal). For example, Voevodsky (2015) and the Univalent Foundations Program (2013) say that it can make quotient types that behave well. It's hard to formalize current mathematics because these ideas aren't part of most type theories.

Cubical Type Theories and Their Models

To make a type theory cubical, two important things must be thought about: the existence of a basic interval I and the theory's ability to include judgment structures that can handle situations with interval variables. These are both very important things that you need to think about. In a way that makes sense, The formal representation of the actual interval is one way to look at it. This occurs as a result of considering the actual time. [0, 1] is a subset of R. Picture the value of the variable i: I as a point that is in constant motion between the two extremes, 0: I and 1: I.

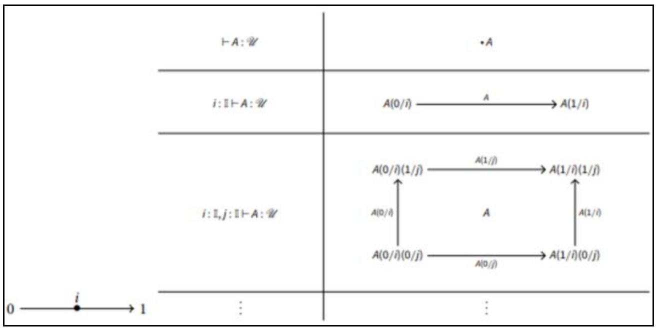

Fig 1: Connection between contexts that include interval variables and cubes is discussed.

Because these factors are added to the MLTT judging structure, we can get judgments in the form of cubes.

$$i_1 : \mathbb{I}, \ldots i_n : \mathbb{I} \vdash \mathcal{J}$$

Given a judgment $\mathcal{J}$ that is dependent on a variable that follows an interval i: II, when we write $\mathcal{J}$ (r/i) for $\mathcal{J}$, we substitute r for i in the text for the letter i. Because of this, the type A(0/i) is a type in which the value 0 has been substituted for i, etc. It is important to note that these changes work the same way with binders as any other type and term formers in the theory. The way they act fits with the idea. This is due to the fact that they perform an identical role to that of regular replacements. There are n-dimensional cubes that correspond to the kinds and phrases that are present in a context that comprises n-dimensional variables. These cubes are described in Figure 1. According to the basic criteria for replacements, we are able to assert that A(0/i) (0/j) is equivalent to A(0/j) (0/i), and other similar statements may be made. Figure 1's third row contains equations that correspond to the lines in the square that are equal to one another. These equations are shown in order. It is important to note that the origin of the line that is located on the leftmost side of the square, denoted by the equation A(0/i), is identical to the origin of the line that is located on the bottom of the square, denoted by the equation A(0/j). Please pay attention to note 1.11 about this subject. It is very important to remember that every judgment could come in the way that was just talked about. Taking into consideration the fact that a judgment Γ⊢ $\mathcal{J}$ in the event when both the regular variables x: and the interval variables i: I am present in the equation. The first thing that we can do is restructure it in such a way that ordinal variables are shown last and interval variables first.

Applications of higher-dimensional types

When it comes to informal mathematics, it is often assumed that all isomorphic groups possess the same qualities as a particular group (in the form of an algebraic structure). Given that one group is abelian, it follows that other isomorphic groups are also abelian. Since the univalence principle literally implies that all isomorphic structures are identical, it is necessary for every attribute to comply to structural equivalence, which is similar to group equivalence. As a result of the fact that there are several equivalences between the two structures, there are also multiple methods that may be used to identify the structures. Using the proof relevance principle, we discovered that different structures have different identification proofs with their own unique characteristics. One of the easiest approaches to defining such a world is to use the process of assigning similarity in structure to the universe's higher-dimensional structures that recognizes structures. To put it another way, the universe is only able to maintain a certain kind of structure via its existence.

We are now going to discuss the patch theorem. In the manner that is shown, patch theories have the potential to be expressed as higher-dimensional versions. Specifically, in the context of this discussion, a patch (for example, "adding a file") refers to an update to the repository and is only applicable under certain patch scenarios (for example, "the file being added does not exist"). However, in addition to that, there are certain formulae, which are known as the patch authorities, that it is hoped that fixes would adhere well. It is possible to combine all of these features of a patch theory into one higher-dimensional type, as shown by Angiuli, Edward Morehouse, Dan Licata, and Robert Harper. This was accomplished by combining all of the patch theory qualities. Patch laws are the rules that describe the patterns of interaction between patch elements and patch laws. Patch elements are the contexts in which patches are implemented, and patch laws are the rules that define those patterns (patterns). It is important to note that a more elegant method to explain irreversible patches is to discard the symmetry of higher-dimensional systems, which is usually considered to be the case. As this article is being written, this particular line of research is still up in the air. When it comes to homotopy theory. These homotopes are easily incorporated into higher-dimensional structures, and the theory of homotopy investigates topological spaces up to the point of continuous deformation (homotopy). When it comes to this particular scenario, we are interested in expressing topo-logical spaces that perform well up to the homotopy equivalence using type theory. When homotropies are internalized as part of types and every element is expected to respect homotropies as a formal proof, a more general theorem is true in a variety of models that also include homotopy structures. The work that I have done for my thesis is likewise based on this fruitful line of investigation.

Remark 1: This method is referred to as synthetic homotopy theory since it does not make use of open sets. This is because synthetic geometry does not make use of coordinates. In its place, new spaces are constructed via the use of fundamental spaces and combinators.

A lot of work has been gone toward automating topological objects that don't internalize homotropies or employ higher-dimensional types. The complete validation of programs like Kenzo, which compute algebraic structures of topological spaces using a number of methodological techniques, might be one conceivable outcome of such findings in the future. While certain topologies, like loop spaces, may need more effort, the resulting proofs can limit the number of usable models. Numbers of quality indicators arranged in a hierarchy. Utilizing higher-dimensional structures allows for the definition of quotients, a crucial mathematics concept. Cauchy real numbers are a famous example; they are also called the quotient of Cauchy sequences. It is often believed that the last rational numbers in a series following a Cauchy pattern are Cauchy reals. This ensures that the numbers of the phrases that follow in the sequence will never diverge by more than the specified distance. Because a correct representation of real numbers has to be characterized in terms of two Cauchy sequences "approaching the same number," this discovery may be classified into higher-dimensional groups. All four of these examples shed light on the notion that higher-dimensional types might have the potential to provide more helpful abstractions for mathematical automation.

Table 1: An essential tool for self-awareness and fair treatment.

	Judgmental equality	**Internalized equality**	**Identification**
Common usage	definitional equality	extensional identity	intentional identity
↳ such as	″	″	″
Common usage	judgmental equality	(not used)	propositional equality
↳ such as	″		and identity
Martin-Löf (1975)	definitional equality	(not used)	identity
Martin-Löf	definitional equality	propositional equality	(not used)
↳ (1982, 1984)	(intensional)	(extensional)	
NuPRL	equality	equality	(not used)

NUPRL nomenclature does not align well with the vocabulary used by others.

The documentation of NUPRL should be personally consulted by readers, since this is highly encouraged.

Conclusion

The integration of logical reasoning, topology, and computational interpretation in a single, unified framework establishes a new foundation for mathematics through differential homotopy theory (HoTT). Unlike classical set theories, which base their identity on extensional equality, HoTT has introduced identification types and higher-dimensional structures that facilitate mathematical interpretation via path, equivalence and homotopy. The existence of a stronger correspondence between mathematical logic and geometric intuition while simultaneously enabling constructive computation are significant results of the introduction of univalent foundations and cubical type theories. The existence of cubical structures gives concrete geometric meaning to equality and makes concrete the realization of univalence in theorem-provers. Thus, mathematical proofs can now be consistently formalised, verified and computed with an increased level of accuracy. These improvements have very much aided in the automation of mathematical reasoning and in furthering proof assistants.

This research further establishes applicable uses of higher dimensional type theory across a variety of academic disciplines. Applications of higher dimensional structures include; synthetic homotopy theory and topology, artificial intelligence (AI), formal verification (FV), and computational mathematics. Higher dimensional structures possess powerful modes of abstraction that can model complex systems of relationships and equivalences. Homotopy Type Theory (HTT) constructs interpretations of mathematical objects and the transformations between them and thus provides a bridge between the abstract theoretical mathematics (topology) and practical computational implementation (AI, FV, and computational mathematics). Therefore, the use of higher dimensional foundations along with cubical homotopy method(s) are seen as major advancements within contemporary mathematical thinking. These same higher dimensional foundational aspects along with cubical homotopy methods provide a theoretical framework for connecting topology, computation, and logic; therefore making HTT one of the most significant frameworks for future innovations within mathematics, computer science, and formal reasoning systems.

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