



An Intuitive Approach to Multiple Centroid Computation for Type-2 Fuzzy Sets

¹Narottam Kumar and ²Dr. Ajay Kumar Singh

^{1,2}Department of Mathematics, Madhyanchal Professional University, Bhopal, Madhya Pradesh, India

DOI: <https://doi.org/10.5281/zenodo.21643607>

Corresponding Author: Narottam Kumar

Abstract

In uncertain decision-making situations, the use of generalized hybrid fuzzy multiple centroids greatly improves the assessment of criteria and alternatives. This method combines a new defuzzification technique called intuitive multiple centroids with the fuzzy TOPSIS algorithm, which ranks solutions according to how close they are to the ideal. In uncertain contexts, traditional decision-making procedures that depend on accurate numerical facts typically fail. Alternatively, Type-2 fuzzy sets provide a more versatile way to mimic the intricacies of human decision-making by acting as useful instruments for approximation reasoning and uncertainty modeling. The fuzziness, ambiguity, and uncertainty that come with human judgments are beyond the capabilities of Type-1 fuzzy sets, which mainly deal with imprecision. This hybrid technique improves decision-making results in uncertain circumstances by combining intuitive multiple centroids with fuzzy TOPSIS. The result is a more robust selection framework.

Keywords: Intuitive, Multiple, Centroid, Computation and Fuzzy

Introduction

Fuzziness and uncertainty are commonplace in many scientific and technical applications; in these fields, challenges involving decision-making often include less-than-clear information with varying degrees of membership grade. Modeling uncertainty or approximation reasoning is a good fit for type-2 fuzzy sets. As a result of its increased power, it is better able to depict the inherent ambiguity in decision-making situations involving humans. Only imprecision, not uncertainty, may be described by type-1 fuzzy sets. One reason type-2 fuzzy sets are gaining popularity is that they provide more room to maneuver when representing uncertainty than type-1 fuzzy sets. They assert that type-2 fuzzy sets are best described as fuzzy membership functions, with membership values in the range $[0,1]$, as opposed to type-1 fuzzy sets, which have crisp values in the interval $[0,1]$. The effectiveness of modeling approaches for fuzzy systems relies heavily on defuzzification. The defuzzification process is governed by the output fuzzy subset, which indicates that a single, crisp value should be chosen as the system output. Although other defuzzification techniques have emerged, much of the research focuses on one particular approach. While each of

these approaches works well in certain contexts, the centroid technique is the only one that consistently delivers good results regardless of the environment.

Fuzzy number centroid defuzzification techniques have been studied and deployed in several fields during the last ten years. Type-2 fuzzy sets are notoriously difficult to implement in real-world settings due to the high computing cost associated with their uncertainty footprint. In order to handle criterion evaluation and alternative evaluation, the hybrid MCDM model integrated two methodologies. An important part of the needs for MCDM approaches is the assessment process of options and criteria. In order to determine which of multiple possible alternatives is the best fit, the approach must first investigate which factors are most valued in order to establish credible weights for use in making the final choice. This study proposes a method for handling uncertainty events that combines consistent fuzzy preference relations with TOPSIS, a fuzzy methodology for ordering preferences by similarity to ideal solutions, utilizing novel centroid defuzzification for interval type-2 fuzzy sets. Classical TOPSIS's main flaws lie in its failure to include weight elicitation and consistency checking into the assessment of judgments.

The suggested model is validated by doing a sensitivity analysis. Since it works on the assumption that a set of weights for criteria or alternatives then got a new set of weights for them, the efficiency of alternatives has become equal or their order has changed, it may effectively help to making correct judgments. A type-2 fuzzy set's centroid may be calculated using type reduction. When using type-2 fuzzy systems, this is a crucial step in the process. The variables in a type-2 fuzzy system are defined using type-2 fuzzy sets, and the system's rules are also of type-2 fuzzy set. The goal is to have the system derive a crisp-valued output from the given inputs. Typically, a type-2 fuzzy set is the final product of combining the inferred conclusions of several rules. As far as anybody can tell, type reduction is an inevitable process that reduces type-2 fuzzy sets to type-1 ones. Defuzzification is then used to transform the type-1 fuzzy set that was produced into a crisp number that was sought. Compared to defuzzification, type reduction often takes a lot more time. Therefore, the increasing interest in using type-2 fuzzy systems in real-world scenarios may benefit from efficient type reduction.

Literature Review

Subjective human judgments, imperfect knowledge, and a variety of contradictory criteria may all be found in real-world circumstances. Such uncertainty cannot be adequately managed by conventional Multiple Criteria Decision-Making (MCDM) techniques. These drawbacks are addressed by MCDM techniques based on fuzzy logic, which simulate uncertainty in expert judgment and include language preferences. This study provides a comprehensive analysis of fuzzy MCDM approaches, from classical fuzzy extensions to more recent breakthroughs in intuitionistic, Pythagorean, and image fuzzy models. Outranking, value-based, pairwise comparison, and hybrid decision models are all included in the study's methodical categorization. Critical examination is done in key application areas including energy planning, healthcare, supply chain management, transportation, and intelligent systems. Issues with subjectivity, computational complexity, model validation, and real-time deployment are discussed. Lastly, future prospects are indicated, such as autonomous decision-making in dynamic situations, data-driven decision support, intelligent automation, and standardization of uncertainty modeling.

A state-of-the-art model for group decision-making using Type-2 fermatean fuzzy sets (T2FFS) with recently introduced Hamming and Euclidean distance metrics is presented in this study. By enhancing and managing ambiguity and uncertainty in expert assessments, the suggested framework overcomes the shortcomings of traditional fuzzy set methods. To circumvent a major drawback of current type-1 and type-2 distance measurements in fuzzy contexts, the distance metrics may faithfully reflect the decision makers' natural levels of doubt and reluctance while evaluating options. An actual issue concerning the selection of electric vehicles (EVs) for environmentally friendly transportation is used to illustrate the model's practical applicability. By comparing it to current methods, we can see that the suggested strategy is

more accurate and reliable.

For difficult decision-making tasks, many fuzzy multiple criteria decision-making (MCDM) models have been suggested. Particularly in the realm of civil engineering, several uses have been realized. This study reviews 52 journal articles that focused on the use of fuzzy MCDM models in civil engineering from 2016 to 2020 in order to assess the current state of the art, identify future research directions, and analyze the recent developments regarding these methods and their applications in the field. Based on the research challenges and approaches discussed in each article, we have categorized them accordingly. The results of the literature study show which decision-making issue is most pressing, which evaluation criteria is most often utilized, and which fuzzy MCDM model is most popular. In addition, we outline four areas of civil engineering research issues and their associated future research paths, which can be useful for academics and professionals looking to delve more into the subject.

In today's highly competitive market, suppliers' environmental performance is of utmost importance. By the way, when it comes to choosing suppliers, organizations have been heavily influenced by economic performance. In this work, we provide a novel integrated model that takes environmental and economic issues into account when evaluating suppliers and allocating orders. We assess suppliers according to environmental standards using interval type-2 fuzzy sets and the EDAS (Evaluation based on Distance from Average Solution) approach. For every vendor, an assessment yields two metrics: a good score and a negative score. In order to determine the order quantity from each supplier, a multi-objective mathematical model is proposed using these factors in addition to cost parameters. This study presents a numerical example to demonstrate the practicality of the integrated model that is suggested. In order to see how different weights for environmental criteria affect the overall cost of purchase and the amount ordered from each supplier, a sensitivity analysis is also conducted. Efficiency and practicality in solving real-world issues are shown by the outcomes of the suggested approach.

For many domains, uncertainty poses a big problem when it comes to data analysis, modeling, and decision-making. The flexibility and interoperability of soft set theory with other models for representing uncertainty (such as fuzzy sets and their extensions) have made it a viable option for dealing with this problem. Slack in the assessments, such as that which results from measurement mistakes, is incompatible with membership-based theories on their own. This problem gave rise to other extensions that enable broader areas of evaluations, such as interval-valued intuitionistic fuzzy sets (with just membership), circular intuitionistic fuzzy sets (with both membership and non-membership), and so on. With this goal in mind, we first integrate circular Pythagorean fuzzy sets with soft set theory. Novel accuracy and scoring systems based on optimistic and pessimistic viewpoints are proposed for this situation. This is accomplished by using a parameter $\mu \in [0,1]$ that is controlled by the decision-maker and represents her/his problem-solving strategy.

Extension of Intuitive Multiple Centroid for Type-2 Fuzzy Sets

Let consider by $\tilde{A} = (\tilde{A}^U, \tilde{A}^L) = ((a_1^U, a_2^U, a_3^U, a_4^U; h_A^U), (a_1^L, a_2^L, a_3^L, a_4^L; h_A^L))$ with the help of a fuzzy interval type-2 number. Following is a representation of the whole procedure for an intuitive multiple-centroid interval type-2 fuzzy set.

Step 1: Find the centroids of the three parts of, $\alpha - a, \underline{\alpha}$, $\beta - \underline{\beta}, \underline{\beta}$, and $\gamma - \underline{\gamma}, \underline{\gamma}$ as seen in Figure 1, inside the interval type-2 fuzzy set. The three components that make up the type-2 fuzzy set for trapezoidal intervals are: 1) two triangles with parallel sides of $\bar{\alpha}$ and $\underline{\alpha}$; 2) two rectangle shapes of $\bar{\beta}$ and $\underline{\beta}$ and; 3) two left triangle shapes of $\bar{\gamma}$ and $\underline{\gamma}$. Triangles, rectangles, and left triangles all have sub centroid values that stand for α_A, β_A as γ_A respectively.

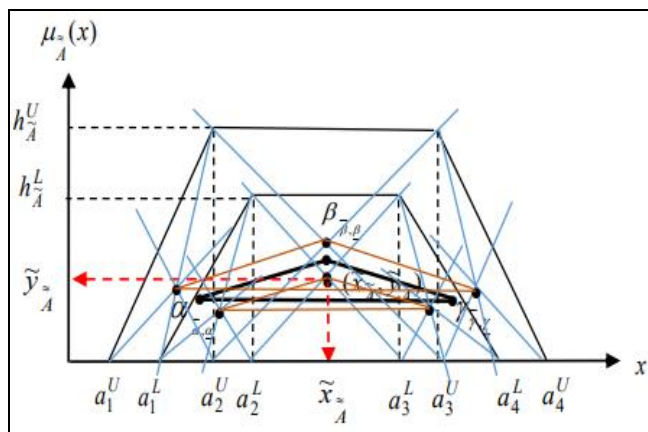


Fig 1: Intuitive multiple centroid plane representation of type-2 fuzzy set

An intuitive multiple centroid defuzzification algorithm should, in theory, choose a median point that adequately covers the shape's center. The median points of are shown in Figures 2, 3, and 4 as distinct sub centroid points $\alpha - a, \underline{\alpha}$, $\beta - \underline{\beta}, \underline{\beta}$, and $\gamma - \underline{\gamma}, \underline{\gamma}$ respectively.

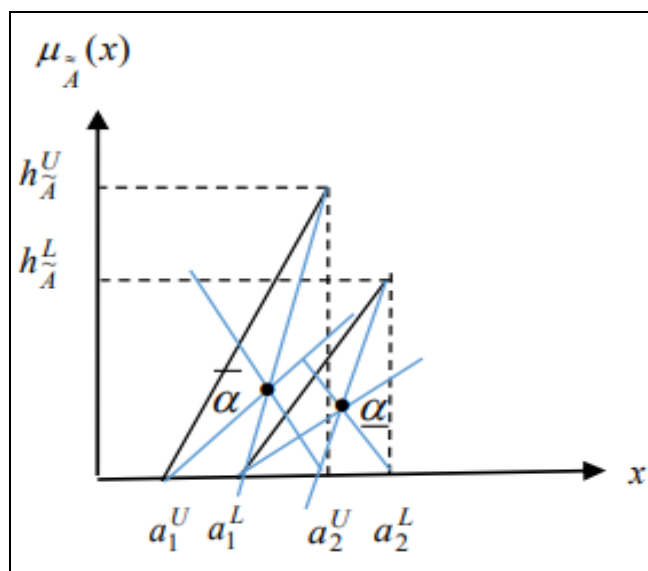


Fig 2: Centroid for upper, $\bar{\alpha}$, and lower forms, α , of left triangles

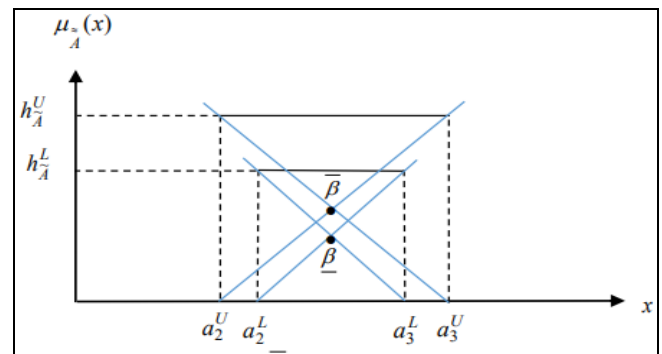


Fig 3: Centroid for upper, $\bar{\beta}$ and lower forms, β , of rectangles

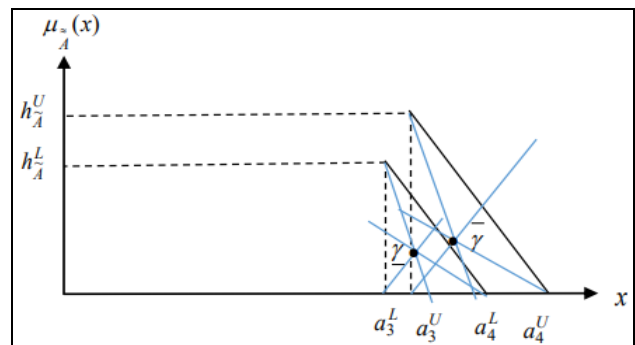


Fig 4: Centroid for upper, $\bar{\gamma}$ and lower forms, γ , of right triangles

Sub centroid points of $\alpha - a, \underline{\alpha}$ formula.

$$\alpha - a, \underline{\alpha}, (\tilde{x}_A, \tilde{y}_A) = \left[\frac{1}{6}(a_1^U + a_1^L) + \frac{1}{3}(a_2^U + a_2^L), \frac{1}{6}(h_A^U + h_A^L) \right] \quad (1.1)$$

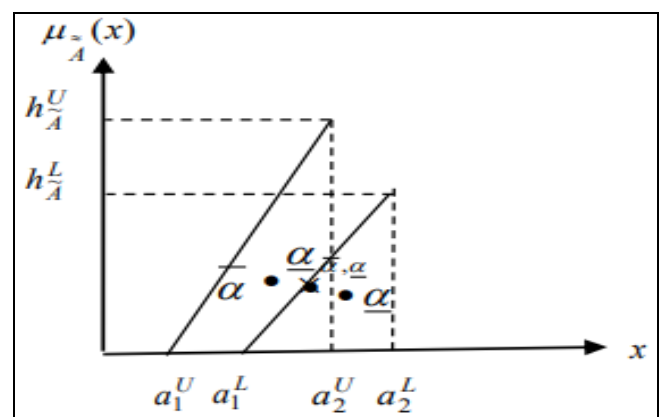


Fig 5: Sub centroid of left triangle, $\alpha - a$

Figure 5 presents $\alpha - a, \underline{\alpha}, (\tilde{x}_A, \tilde{y}_A)$ sub centroid point of $\alpha - a, \underline{\alpha}$, left shape of triangle are developed. Basically, the sub centroid for $\alpha - a, \underline{\alpha}$. A α_A for fuzzy sets of type 1. Fuzzy numbers of interval type 2 have higher, $\bar{\alpha}$ and lower, $\underline{\alpha}$ form, whereby it is represented on a plane with two type-1 fuzzy numbers. Most researchers in the literature simply locate the midway between the upper and lower forms when trying to derive the centroid for interval type-2 fuzzy sets. To find the midpoints between, the expansion of the suggested intuitive multiple centroids uses the same procedure $\bar{\alpha}$ and $\underline{\alpha}$. The midpoints between are defined by Equation (1.2) $\bar{\alpha}$ and $\underline{\alpha}$.

$$\bar{\alpha}(x_{\tilde{A}}, y_{\tilde{A}}) = \left[a_1^U + \left[\frac{2}{3} (a_2^U - a_1^U) \right], \frac{h_{\tilde{A}}^U}{3} \right]$$

$$\underline{\alpha}(x_{\tilde{A}}, y_{\tilde{A}}) = \left[a_1^L + \left[\frac{2}{3} (a_2^L - a_1^L) \right], \frac{h_{\tilde{A}}^L}{3} \right]$$

Then,

$$a_{-\alpha, \alpha}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{(a_1^U + [\frac{2}{3}(a_2^U - a_1^U)]) + (a_1^L + [\frac{2}{3}(a_2^L - a_1^L)])}{2}, \frac{\frac{h_{\tilde{A}}^U}{3} + \frac{h_{\tilde{A}}^L}{3}}{2} \right]$$

$$a_{-\alpha, \alpha}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{(a_1^U + \frac{2a_2^U - 2a_1^U}{3}) + (a_1^L + \frac{2a_2^L - 2a_1^L}{3})}{2}, \frac{h_{\tilde{A}}^U}{6} + \frac{h_{\tilde{A}}^L}{6} \right]$$

$$a_{-\alpha, \alpha}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{(\frac{a_1^U}{3} + \frac{2a_2^U}{3}) + (\frac{a_1^L}{3} + \frac{2a_2^L}{3})}{2}, \frac{1}{6}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right]$$

$$a_{-\alpha, \alpha}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\left(\frac{a_1^U}{6} + \frac{a_1^L}{6} \right) + \left(\frac{2a_2^U}{6} + \frac{2a_2^L}{6} \right), \frac{1}{6}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right]$$

Hence,

$$a_{-\alpha, \alpha}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{1}{6}(a_1^U + a_1^L) + \frac{1}{3}(a_2^U + a_2^L), \frac{1}{6}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right]$$

Sub centroid points of $\beta - \beta, \beta$ formula.

$$\gamma_{\beta, \beta}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left(\frac{1}{6}(a_1^U + a_1^L) + \frac{1}{3}(a_2^U + a_2^L), \frac{1}{6}(h_{\tilde{A}}^U + h_{\tilde{A}}^L) \right) \quad (1.2)$$

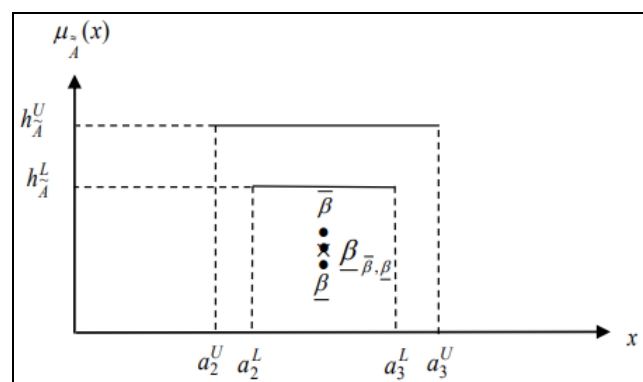


Fig 6: Sub centroid of rectangle, $\beta - \beta, \beta$

Same goes to $\beta - \beta, \beta(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$, the middle point between $\bar{\beta}$ and β is computed. Fundamentally, the sub centroid for $\beta - \beta, \beta(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ is identical to the idea, $\bar{\alpha}$ for fuzzy sets of type 1. The midpoints between are defined. $\bar{\alpha}$ and $\underline{\alpha}$:

$$\beta - \beta, \beta(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{(\frac{a_2^U + a_3^U}{2}) + (\frac{a_2^L + a_3^L}{2})}{2}, \frac{\frac{h_{\tilde{A}}^U}{2} + \frac{h_{\tilde{A}}^L}{2}}{2} \right]$$

$$\beta_{\bar{\beta}, \bar{\beta}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\left(\frac{a_2^U + a_3^U}{4} \right) + \left(\frac{a_2^L + a_3^L}{4} \right), \left(\frac{h_A^U}{4} + \frac{h_A^L}{4} \right) \right]$$

Hence,

$$\beta_{\bar{\beta}, \bar{\beta}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{1}{4}(a_2^U + a_3^U + a_2^L + a_3^L), \frac{1}{4}(h_A^U + h_A^L) \right]$$

Sub centroid points of $\gamma_{\bar{\gamma}, \gamma}$ formula.

$$\gamma_{\bar{\gamma}, \gamma}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \left[\frac{1}{6}(a_4^U + a_4^L) + \frac{1}{3}(a_3^U + a_3^L), \frac{1}{6}(h_A^U + h_A^L) \right] \quad (1.3)$$

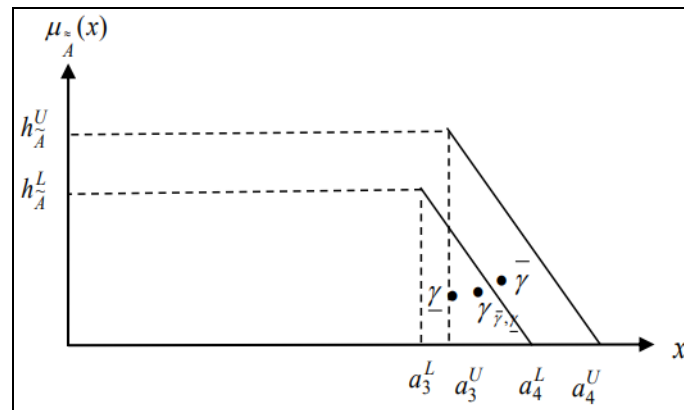


Fig 7: Sub centroid of right triangle, $\gamma_{\bar{\gamma}, \gamma}$

The formulation created for is shown in Figure 7. $\gamma_{\bar{\gamma}, \gamma}(x_{\tilde{A}}, y_{\tilde{A}})$ sub centroid point of $\gamma_{\bar{\gamma}, \gamma}$ triangular form on the left side. Equation (1.5)'s solution is derived in a manner similar to that of the sub centroid points of $\alpha_{\bar{\alpha}, \alpha}$.

Step 2: Join the points on each vertex that are the centers of $\alpha_{\bar{\alpha}, \alpha}$, $\beta_{\bar{\beta}, \beta}$ and $\gamma_{\bar{\gamma}, \gamma}$ opposite one another, forming an additional triangle plane inside the trapezoid plane.

Step 3: Intuitive multiple centroids' centroid index $(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}})$ with vertices, $\alpha_{\bar{\alpha}, \alpha}$, $\beta_{\bar{\beta}, \beta}$ and $\gamma_{\bar{\gamma}, \gamma}$ can be calculated as $IMC_{\tilde{A}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) =$

$$\left[\beta_{\bar{\beta}, \bar{\beta}}(\tilde{x}_{\tilde{A}}) + \frac{2}{3} \left(\frac{\alpha_{\bar{\alpha}, \alpha}(\tilde{x}_{\tilde{A}}) + \gamma_{\bar{\gamma}, \gamma}(\tilde{x}_{\tilde{A}})}{2} - \beta_{\bar{\beta}, \bar{\beta}}(\tilde{x}_{\tilde{A}}) \right), \beta_{\bar{\beta}, \bar{\beta}}(\tilde{y}_{\tilde{A}}) + \frac{2}{3} \left(\frac{\alpha_{\bar{\alpha}, \alpha}(\tilde{y}_{\tilde{A}}) + \gamma_{\bar{\gamma}, \gamma}(\tilde{y}_{\tilde{A}})}{2} - \beta_{\bar{\beta}, \bar{\beta}}(\tilde{y}_{\tilde{A}}) \right) \right] \quad (1.4)$$

Multiple centroids based on intuition may be described as

$$IMC_{\tilde{A}}(\tilde{x}_{\tilde{A}}, \tilde{y}_{\tilde{A}}) = \frac{2(a_1^U + a_1^L + a_4^U + a_4^L) + 7(a_2^U + a_2^L + a_3^U + a_3^L)}{36}, \frac{7}{36}(h_A^U + h_A^L) \quad (1.5)$$

where,

$\alpha_{\bar{\alpha}, \alpha}$: where the first triangle's centroid is located on the plane

$\beta_{\bar{\beta}, \bar{\beta}}$: the coordinates of the rectangle's centroid on the plane

$\gamma_{\bar{\gamma}, \gamma}$: the second triangle's plane centroid

$(x_{\tilde{A}}, y_{\tilde{A}})$: the location of the fuzzy number's centroid $\tilde{A}$

Conclusion

Fuzzy TOPSIS's incorporation of intuitive multiple

centroids is a huge step forward in handling the intricacies of human-based decision-making. Particularly when confronted with massive volumes of data, the inherent fuzziness, ambiguity, and unpredictability of real-world situations may be a formidable challenge for conventional decision-making frameworks. More complex and fruitful decisions are possible with the help of these hybrid techniques, which include intuitive reasoning and recognize human subjectivity. Given that traditional approaches struggle to deal with uncertainty and imprecision, type-2 fuzzy systems become an effective tool in this setting. In

complicated decision-making contexts, type-2 fuzzy sets shine because they allow approximate reasoning, in contrast to conventional systems that depend on exact numerical data. Important for extracting useful information from these systems is the defuzzification procedure, which converts fuzzy results into sharp numbers. Despite the fact that several defuzzification approaches have been investigated in the literature, type-2 fuzzy systems will continue to be improved via the continuous research and improvement of these techniques.

References

1. Wen Z, Zavadskas EK, Antucheviciene J. Applications of fuzzy multiple criteria decision-making methods in civil engineering: A state-of-the-art survey. *Journal of Civil Engineering and Management*. 2021;27(6):358-371. doi:10.3846/jcem.2021.15252.
2. Keshavarz-Ghorabae M, Amiri M, Zavadskas EK, Turskis Z, Antucheviciene J. A new multi-criteria model based on interval type-2 fuzzy sets and EDAS method for supplier evaluation and order allocation with environmental considerations. *Computers and Industrial Engineering*. 2017;112:660-674. doi:10.1016/j.cie.2017.08.017.
3. Pedrycz W, Ekel P, Parreiras RO. *Fuzzy Multicriteria Decision-Making: Models, Methods and Applications*. Hoboken (NJ): John Wiley & Sons; 2011. doi:10.1002/9780470974032.
4. Dadasheva A. Multicriteria group decision making on information system project selection using type-2 fuzzy set. In: *Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation*. Cham: Springer; 2023. p. 219-230. doi:10.1007/978-3-031-25252-5_19.
5. Abdullah L. Fuzzy multi-criteria decision making and its applications: A brief review of category. *Procedia - Social and Behavioral Sciences*. 2013;97:131-136. doi:10.1016/j.sbspro.2013.10.213.
6. Chen L, Liu W, Zhang J. An interval type-2 fuzzy multi-criteria decision-making approach for patient bed allocation. *Scientific Reports*. 2025;15:Article 9199. doi:10.1038/s41598-025-09199-1.
7. Naim S, Hagrass H. A type-2 hesitation fuzzy logic-based multi-criteria group decision-making system for intelligent shared environments. *Soft Computing*. 2014;18(7):1305-1321. doi:10.1007/s00500-013-1145-0.
8. Ozdemir Y. A spherical fuzzy multi-criteria decision-making model for Industry 4.0 performance measurement. *Axioms*. 2022;11(7):325. doi:10.3390/axioms11070325.
9. Özlü Ş. Multi-criteria decision making based on vector similarity measures of picture type-2 hesitant fuzzy sets. *Granular Computing*. 2023;8:1241-1257. doi:10.1007/s41066-023-00382-1.
10. Kansal D, Kumar S. Multi-criteria decision-making based on intuitionistic fuzzy exponential knowledge and similarity measure and improved VIKOR method. *Granular Computing*. 2024;9:1-22. doi:10.1007/s41066-023-00448-0.

This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY 4.0) license. This license permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.