E-ISSN: 2583-9667
Indexed Journal
Peer Reviewed Journal
https://multiresearchjournal.theviews.in

Received: 01-10-2025
Accepted: 05-11-2025
Published: 06-01-2026

INTERNATIONAL JOURNAL OF ADVANCE RESEARCH IN MULTIDISCIPLINARY

Volume 4; Issue 1; 2026; Page No. 295-299

A Rank-Assisted Intuitionistic Fuzzy Approach to Unbalanced Transportation Problems with Triangular Intuitionistic Fuzzy Numbers

[1]Lokande Ravindar and [2]Dr. Lokendra Kumar

[1]Research Scholar, Department of Mathematics, Monad University, Hapur, Uttar Pradesh, India
[2]Assistant Professor, Department of Mathematics, Monad University, Hapur, Uttar Pradesh, India

DOI: https://doi.org/10.5281/zenodo.21898690

Corresponding Author: Lokande Ravindar

Abstract

Unbalanced transportation problems become more difficult when costs, capacities and requirements are imprecise and when available evidence contains both support and rejection. This paper presents a rank-assisted solution framework based on triangular intuitionistic fuzzy numbers (TIFNs). Each uncertain parameter is represented by a lower, modal and upper value together with membership and non-membership grades. A centroid-like magnitude is used for operational comparison, while score and hesitation provide secondary information. Supply-demand imbalance is corrected through a dummy destination or source whose economic interpretation is made explicit. A direct zero-suffix procedure constructs the initial basis and the Modified Distribution Method (MODI) supplies the optimality certificate. In a surplus-supply example, the direct basis has representative cost 535; one MODI loop reduces the cost to 525, and reconstruction with the original TIFN route costs gives the intuitionistic fuzzy objective <(460, 525, 590); 0.90, 0.05>. A shortage example requires a 10-unit dummy source with destination-specific penalties and yields a representative optimum of 595, with the dummy allocation identifying the destination exposed to shortage or emergency procurement. The framework demonstrates that ranking, balancing, allocation, optimality and uncertainty interpretation should be separated rather than merged into one opaque calculation.

Keywords: Unbalanced transportation, intuitionistic fuzzy set, triangular intuitionistic fuzzy number, zero suffix, MODI, dummy penalty, uncertainty

1. Introduction

Transportation models are frequently used to allocate a homogeneous product from several sources to several destinations at minimum total cost. The conventional formulation assumes exact costs, supplies and demands. In practice, however, freight prices may fluctuate, usable capacity may be uncertain and demand forecasts may contain disagreement among experts. Fuzzy transportation models address imprecision by replacing exact parameters with fuzzy numbers. Intuitionistic fuzzy sets extend this idea by recording not only membership but also non-membership, leaving a separate hesitation component for information that is neither fully supportive nor fully contradictory (Atanassov, 1986) [1].

The distinction is valuable when lack of support should not automatically be interpreted as evidence against a value. For example, a route-cost estimate may be supported by recent invoices but contradicted by fuel-price volatility, while part of the evidence remains unresolved. A triangular intuitionistic fuzzy number can preserve these dimensions through a lower, modal and upper value together with membership and non-membership grades. The challenge is to translate that richer uncertainty description into a reproducible transportation decision without pretending that the uncertain parameter is already crisp.

Unbalanced problems add another modelling issue. When total supply exceeds total demand, the surplus can represent unused capacity, inventory or disposal. When demand exceeds supply, the shortage can represent unmet demand, emergency procurement or subcontracting. The standard dummy source or destination remains mathematically useful, but its cost and interpretation matter. A zero-cost dummy source, for instance, would make shortage artificially free. This paper therefore integrates TIFN representation, explicit dummy construction, a zero-suffix initial basis, MODI verification and reconstruction of the intuitionistic fuzzy objective.

2. Triangular Intuitionistic Fuzzy Representation

A triangular intuitionistic fuzzy number is defined by a numerical triple and two evidence grades. The representation used here is

$$\tilde{A}^I = \langle (a, b, c); w, u \rangle, a \leq b \leq c, w + u \leq 1 \quad (1)$$

where a, b and c are the lower, modal and upper numerical values, w is the maximum membership grade and u is the minimum non-membership grade. The requirement w + u <= 1 leaves room for hesitation. At any point x, the hesitation degree is

$$\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x) \quad (2)$$

Membership, non-membership and hesitation should not be interpreted as probabilities. They are graded descriptions of evidence under an intuitionistic fuzzy model. This distinction is particularly important when uncertainty is elicited from expert judgement or incomplete operational records rather than estimated from a statistical sampling distribution.

The operational comparison rule uses a centroid-like magnitude C. For equal centroids, the net support score S and the hesitation at the modal point can be used as secondary criteria.

$$C(\tilde{A}^I) = \frac{a + 2b + c}{4}, S(\tilde{A}^I) = w - u, \Gamma(\tilde{A}^I) = 1 - w - u \quad (3)$$

For symmetric TIFNs of the form <(c-1, c, c+1); w, u>, the centroid is exactly c. This property is convenient in numerical illustrations because uncertainty can be added around a central route cost without changing the representative order. The ranking remains a declared modelling choice rather than a universal ordering theorem. Other score, accuracy or distance rules could be substituted and compared through sensitivity analysis (Li *et al.*, 2010) [10].

3. Unbalanced Intuitionistic Fuzzy Transportation Model

Let x_ij denote the non-negative quantity transported from source i to destination j. Each route cost, source quantity and destination quantity is represented by a TIFN. The framework separates two layers. The uncertain layer retains all original TIFN parameters. The operational layer maps them to representative numerical values for balance, allocation and optimality calculations. This makes the analysis auditable: a change in ranking can be tested without changing the underlying uncertain data.

Representative total supply and demand are calculated by summing centroid magnitudes.

$$A_R = \sum_i C(\tilde{a}_i^I), \qquad B_R = \sum_j C(\tilde{b}_j^I) \quad (4)$$

If A_R > B_R, a dummy destination with representative demand A_R - B_R is introduced. If B_R > A_R, a dummy source of B_R - A_R is required. A zero dummy cost is defensible only when the balancing quantity genuinely carries no relevant economic consequence. For shortage, the dummy source should normally contain an emergency procurement or unmet-service penalty. For surplus, a non-zero dummy destination may be required when unused production must be stored, scrapped or carried forward.

4. Rank-Assisted Zero-Suffix Solution Procedure

The direct allocation stage begins by converting each TIFN cost into its representative magnitude. The active matrix is reduced by subtracting row minima and then column minima. Every active row and column consequently contains at least one zero. For each zero cell, a suffix is calculated from the nearest strictly positive reduced alternatives in the horizontal and vertical directions. A larger suffix indicates a larger local opportunity cost associated with not using that zero route.

The zero with the largest suffix is chosen. Suffix ties are resolved by the largest feasible immediate allocation, then by the smaller original representative cost, stronger intuitionistic support and fixed position. The allocated quantity is the smaller of residual supply and residual demand.

$$x_{ij}^* = \min\{a_i^{(r)}, b_j^{(r)}\} \quad (5)$$

Rows and columns whose residual quantities reach zero are removed and the reduced table is recalculated. The procedure therefore terminates after a finite number of positive allocations, provided the balanced network remains connected through admissible routes. The zero-suffix stage is treated as a basis generator, not as an independent theorem of global optimality.

After a feasible basis is obtained, MODI potentials are fitted to the original representative costs of the basic cells. Every non-basic reduced cost is then tested. The ranked plan is optimal when

$$\Delta_{ij} = r_{ij} - (u_i + v_j), \ \Delta_{ij} \geq 0 \quad \forall (i,j) \notin \mathcal{B} \quad (6)$$

A negative reduced cost identifies an entering route. A closed plus-minus loop then shifts the largest feasible amount that preserves non-negativity. This division between constructive allocation and exact optimality verification avoids an important weakness of many direct heuristics: stopping because the table is complete does not by itself prove that the result is minimum-cost.

5. Numerical Example I: Surplus Supply

Consider three sources with representative supplies 20, 30 and 25, giving total supply 75. Four real destinations require 10, 15, 20 and 20 units, giving total demand 65. A dummy destination of 10 units is therefore added. Unused capacity is assumed to have no transportation cost in this illustration, so the dummy route has the exact intuitionistic fuzzy zero <(0,0,0); 1,0>.

Each real route cost with centre c is represented as <(c-1,c,c+1); 0.90,0.05>. Each source quantity a is <(a-2,a,a+2); 0.92,0.04>, while each real demand b is <(b-1,b,b+1); 0.92,0.04>. The aggregate supply support is (69,75,81) and the real-demand support is (61,65,69), giving an ordered dummy-demand support (8,10,12) with centroid 10.

Table 1: Representative cost table for the surplus example

Source	D1	D2	D3	D4	D5 Dummy	Supply
S1	8	6	10	9	0	20
S2	9	12	13	7	0	30
S3	14	9	16	5	0	25
Demand	10	15	20	20	10	75

The direct zero-suffix procedure generates a feasible basis with representative cost 535. MODI potentials reveal one negative reduced cost, -2 on route S2-D5. The improvement loop is S2-D5(+), S3-D5(-), S3-D4(+) and S2-D4(-). The smallest shipment on the negative positions is 5 units. After the loop adjustment, the objective falls by 10 to 525. A second MODI calculation contains no negative reduced cost, so the revised plan is representative-cost optimal.

Table 2: Ranked-optimal allocation after MODI improvement

Source	D1	D2	D3	D4	D5	Supply
S1	0	15	5	0	0	20
S2	10	0	15	0	5	30
S3	0	0	0	20	5	25
Demand	10	15	20	20	10	75

The representative objective is 15(6) + 5(10) + 10(9) + 15(13) + 5(0) + 20(5) + 5(0) = 525. Ten units are assigned to the dummy destination, so 65 units move to real destinations.

6. Reconstruction of the Intuitionistic Fuzzy Objective

The final shipment matrix is evaluated using the original TIFN route costs rather than the reduced or ranked matrix. For positive shipments, the intuitionistic fuzzy route contributions are added component wise, while the aggregate membership height is taken conservatively as the minimum and the aggregate non-membership floor as the maximum.

$$\tilde{Z}^I = \oplus_{i,j:x_{ij}>0} x_{ij}\tilde{c}_{ij}^I = \langle (Z_L, Z_M, Z_U); w_Z, u_Z \rangle \quad (7)$$

Every real selected route in the example has a unit support one below and one above its modal cost. Since 65 units are moved on real routes, the lower objective is 525 - 65 = 460 and the upper objective is 525 + 65 = 590. The aggregate membership and non-membership values are 0.90 and 0.05. Hence the final intuitionistic fuzzy transportation cost is <(460,525,590); 0.90,0.05>. The modal hesitation is 1 - 0.90 - 0.05 = 0.05.

Table 3: Final intuitionistic fuzzy objective

Measure	Value
Lower cost ZL	460
Modal cost ZM	525
Upper cost ZU	590
Membership height	0.90
Non-membership floor	0.05
Modal hesitation	0.05

The result shows why the representative optimum should not be reported alone. The value 525 is the central operational cost used for route optimisation, whereas the support from 460 to 590 records imprecision inherited from the selected route costs. It is not a confidence interval. Likewise, hesitation is not a probability of failure; it is the uncommitted part of the intuitionistic fuzzy evidence.

7. Numerical Example II: Demand Exceeds Supply

A second example illustrates shortage. Three physical sources have representative supplies 20, 25 and 15, giving 60 units, while four destinations require 15, 10, 20 and 25 units, giving 70. A dummy source of 10 units is therefore necessary. Unlike the zero-cost surplus dummy in the first example, this dummy represents emergency procurement or service shortage and is assigned destination-specific penalties.

Table 4: Representative shortage model and dummy penalties

Source	D1	D2	D3	D4	Supply
S1	7	9	8	11	20
S2	10	6	12	9	25
S3	8	13	7	5	15
Dummy source	18	20	17	22	10
Demand	15	10	20	25	70

One ranked-optimal plan sends 15 units from S1 to D1 and 5 to D3; 10 units from S2 to D2 and 15 to D4; 5 units from S3 to D3 and 10 to D4; and all 10 dummy units to D3. The representative total cost is 595. The dummy allocation is substantive information: it identifies D3 as the destination at which shortage or emergency sourcing is economically least costly under the stated penalties. If the dummy source is interpreted as unmet demand rather than external procurement, the same 10 units should be reported as a service shortfall and not as a physical shipment.

8. Sensitivity and Decision Interpretation

The framework is intentionally modular. The primary centroid rule is transparent, but it does not exhaust the uncertainty information. A decision-maker who is averse to wide cost supports or high hesitation can introduce a risk-adjusted representative cost that adds penalties for spread and hesitation.

$$P_{ij} = C(\tilde{c}_{ij}^I) + \lambda(c_{ij}^U - c_{ij}^L) + \eta\Gamma(\tilde{c}_{ij}^I) \quad (8)$$

Here lambda measures aversion to imprecision and eta measures aversion to hesitation. Setting both coefficients to zero returns the centroid model. Positive values can change row minima, zero positions and the final basis, so the adjusted ranking should be treated as a declared sensitivity scenario rather than inserted without justification. A useful robustness report records whether the same basic routes persist across several defensible ranking rules.

Dummy costs require similar sensitivity analysis. A surplus dummy should not automatically be free if unused output creates storage, disposal or opportunity costs. A shortage dummy should reflect the consequence of external procurement, back-ordering, lost sales or service failure. The penalty should be sourced from the application rather than selected merely to force a preferred allocation. Destination-specific penalties can be especially important because shortage consequences are rarely identical across customers.

The numerical examples also show the value of independent verification. In the surplus case, the direct zero-suffix basis was close to the optimum but not yet optimal; one MODI loop reduced the representative cost from 535 to 525. Without the reduced-cost check, the direct result could have been reported incorrectly as final. This distinction between a good constructive basis and a certified optimum is central to reproducible transportation research.

The main limitation is the ranking step itself. Any scalar mapping compresses a multidimensional intuitionistic fuzzy description. Different rankings may reverse two overlapping TIFNs, especially when their central values are close but their evidence grades or spreads differ. The appropriate response is not to claim that one ranking is universally correct, but to state its interpretation, preserve the original TIFNs and examine basis stability under plausible alternatives. For high-stakes logistics applications, the method should also be compared with exact linear-programming or network-flow software and extended to route capacities, multiple commodities or robust constraints where required.

8.1 Parameter elicitation, robustness and implementation

Practical use of the model depends on disciplined construction of the TIFNs. A central route cost can be obtained from recent invoices, contracted rates or an engineering estimate, while lower and upper values can reflect observed variation, scenario limits or expert ranges. Membership should record the strength of support for the triangular description and non-membership should represent separate evidence against it. Assigning the same evidence grades to every route is useful for a transparent numerical illustration, but real applications should permit heterogeneous grades where data quality differs. Otherwise the intuitionistic layer adds notation without adding decision information.

The same principle applies to supply and demand. Capacity uncertainty may arise from yield, maintenance or procurement reliability, whereas demand uncertainty may arise from forecast error or customer behaviour. These mechanisms should not be collapsed automatically into one common spread. Documenting how each parameter was elicited allows later users to understand whether a wide support reflects genuine volatility, limited evidence or conservative judgement. It also permits sensitivity analysis to focus on the parameters that are most influential for the transportation basis.

A robust computational implementation should preserve the original TIFN table as immutable input and create a separate representative matrix for each ranking scenario. The allocation and MODI stages then operate on the representative matrix, while the completed shipment plan is returned to the original uncertain layer for objective reconstruction. This architecture makes it possible to compare centroid ranking, score-based ranking or risk-adjusted ranking without accidentally changing the source data. It also supports route-stability analysis: a route can be reported as stable when it remains basic across a defined set of plausible ranking scenarios.

The dummy structure should be handled in the same transparent way. A dummy route is not merely a mathematical zero added to square the table. Its quantity and penalty can encode unused capacity, inventory, emergency supply, lost sales or service failure. A useful sensitivity study varies the dummy penalty across a defensible range and identifies the thresholds at which the location of surplus or shortage changes. This information can be more valuable to a manager than a single optimum because it shows which destinations become vulnerable when the economic consequence of shortage is reassessed.

For larger networks, the direct zero-suffix stage can be retained as an interpretable basis generator and passed to a standard transportation or linear-programming solver. The solver then supplies the exact optimum, while the intuitionistic fuzzy layer continues to provide uncertainty interpretation. This hybrid design separates the tasks that each component performs well: zero-based reasoning explains route priority, exact optimisation certifies cost efficiency, and TIFNs communicate imprecision and unresolved evidence.

The reporting stage should mirror this separation. A complete result should present the representative shipment matrix, representative objective, dummy allocation, reconstructed intuitionistic fuzzy objective and the ranking assumptions used to obtain them. Where the solution changes under a plausible alternative ranking or dummy penalty, the competing plan should be reported rather than suppressed. Such disclosure makes the uncertainty analysis useful for decision support because it distinguishes a mathematically stable recommendation from one that depends materially on subjective parameter choices.

In addition, future empirical applications should record computation time and post-allocation pivots alongside objective values. These measures would show whether the direct basis contributes practical efficiency as network size grows and would separate mathematical quality from presentation convenience.

9. Conclusion

This paper formulated a rank-assisted approach for unbalanced transportation problems with triangular intuitionistic fuzzy parameters. The method retains membership, non-membership and hesitation in the data layer, uses a centroid-based representative layer for balance and allocation, constructs a feasible basis through zero-suffix reasoning and verifies optimality by MODI. In the surplus example, one post-allocation improvement reduced the representative cost from 535 to the optimum 525, and the original TIFN costs produced the final objective $\langle(460,525,590); 0.90,0.05\rangle$. In the shortage example, a 10-unit dummy source with explicit destination penalties produced an optimum of 595 and located the shortage at D3. The central contribution of the framework is procedural clarity: uncertainty representation, ranking, dummy interpretation, basis construction, optimality and fuzzy reconstruction are treated as separate, auditable stages. This makes the method suitable as a reproducible baseline for further work on alternative intuitionistic rankings, risk-adjusted costs, multi-objective transportation and real logistics applications.

10. References

1. Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986;20:87-96.

2. Bellman RE, Zadeh LA. Decision-making in a fuzzy environment. Management Science. 1970;17:B141-B164.
3. Chanas S, Kolodziejczyk W, Machaj AA. A fuzzy approach to the transportation problem. Fuzzy Sets and Systems. 1984;13:211-221.
4. Chanas S, Kuchta D. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Sets and Systems. 1996;82:299-305.
5. Dantzig GB. Linear programming and extensions. Princeton (NJ): Princeton University Press; 1963.
6. Dubois D, Prade H. Operations on fuzzy numbers. International Journal of Systems Science. 1978;9:613-626.
7. Ebrahimnejad A, Verdegay JL. An efficient computational approach for solving type-2 intuitionistic fuzzy numbers based transportation problems. International Journal of Computational Intelligence Systems. 2016;9:1154-1173.
8. Kaur A, Kumar A. A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. Applied Soft Computing. 2012;12:1201-1213.
9. Kumar PS, Hussain RJ. A method for solving unbalanced intuitionistic fuzzy transportation problems. Notes on Intuitionistic Fuzzy Sets. 2015;21(3):54-65.
10. Li DF, Nan JX, Zhang MJ. A ranking method of triangular intuitionistic fuzzy numbers and application to decision making. International Journal of Computational Intelligence Systems. 2010;3:522-530.
11. Samuel AE, Raja P, Thota S. A new technique for solving unbalanced intuitionistic fuzzy transportation problems. Applied Mathematics & Information Sciences. 2020;14(3):459-465.
12. Zadeh LA. Fuzzy sets. Information and Control. 1965;8:338-353.
13. Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems. 1978;1:45-55.